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A survey was made to study the discrepancy between adult students' mathematical abilities and their opinions of these abilities. A test was composed which comprised 40 questions spanning mathematics from 10th year to first year college level; together with a questionnaire, it was sent to participants in adult education classes in physical and biological subjects and in nonscience classes, in city, suburban, and county areas. A small control group was drawn from the staff of Senate House, administrative headquarters of London University. The parameters derived from total scores were Residual Mathematical Age (RMA) and Retention. Women adult students were found inferior in Retention, although the younger women in the control group did not show inferiority. Retention was inversely related to education; time since mathematical education ended was of surprisingly little significance; and people retain more mathematics than they think they do. Distributions of RMA and Retention were strongly dependent on the type of course attended; people competent in mathematics appeared in physical science courses and absent from nonscience courses; women did not join classes demanding mathematical ability unless they were far above average. [Not available in hardcopy due to marginal legibility of original document.] (pt)

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# Residual Mathematics in Adults



Harry Frost

# RESIDUAL MATHEMATICS IN ADULTS

a study of retention into adult life of
mathematics taught at school:
with particular reference to students in adult education

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the study was carried out in 1962
& published in this form in 1967

## CONTENTS

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## 1. Attitudes to Mathematics

No topic, except the mother tongue, is more certain than mathematics to be included in formal education and to be practised afterwards - even if only at the level of domestic accounts. Yet ignorance and incompetence are admitted, or even proudly claimed, more readily with mathematics than with any other subject of study. "I never had a geographical mind" or "I was always held back by my history" are rare, even from speakers whose insight is far from obvious; similar statements about mathematics are common.

In adult education, especially in science courses for non-specialists, a tutor sometimes wants to use a mathematical concept which, from their formal education, his students might be expected to understand. If he asks in advance whether they do understand it, they almost always say "no". If he uses it without comment they follow the argument and often raise sophisticated questions in discussion.

The present enquiry was initially undertaken to study this discrepancy between adult students' mathematical abilities and adult students' opinions of their mathematical abilities, as an aid to the planning of adult science courses, and its conclusions have now been used for that purpose for nearly five years. This account has now been prepared in order to publish some wider implications - for mathematics education in fields other than adult education and for adult education in fields other than mathematics. The chapter headings indicate where readers, mainly interested in only one of these aspects, should give more or less close attention.

## 2. Outline of Survey

The enquiry involved the setting of a test-paper in mathematics to the largest and most diverse sample of adults that was practicable. Nine-tenths of the 'sample' was drawn from adult education classes, not only because of the initial concern of the enquiry with adult education, but also as a matter of necessity. It is only necessary to consider the probable fate of an investigator who called at every tenth house, with an examination paper in mathematics to recognise that choice of sample for an enquiry of this sort is not unrestricted!

The guinea-pigs had to be co-operative to the point of giving a good deal of time and effort. They might have been sought in many ways, but people who, for example, replied to some sort of public advertisement would certainly have been a-typical. The realistic course was to work with captive groups where a little pressure was possible to bring in marginally co-operative members; there appeared to be no hope of including the unco-operative.

Although a sample from adult courses could be expected to include a reasonable spread of temperaments and abilities, it would never be a representative cross-section of the population. Semi-skilled and unskilled workers are scarcely to be found in adult education, and skilled manual and routine non-manual workers are uncommon; in all, something like half the population - the less intellectual part - must be unrepresented.

For present purposes this limitation is not hard to accept because the people who are not represented seem unlikely, in the foreseeable future, to want to use mathematics or to find out about it. They represent problems in modern society outside the scope of the enquiry.

We do not know how far adult students are typical even of those parts of society for which mathematics might be a consideration. Students are a self-selected minority. They are also selected by the programmes of courses offered; these are neither completely random in their coverage nor designed to provide a representative sample of the subjects which <u>might</u> be offered.

Finally, when students in courses are approached to take part in the survey, it remains, for all the pressure applied by tutors and lecturers, a self-selected portion which responds.

The best that could be said, a priori, about the sample is that there seemed no way of finding a better. A posteriori, it turned out to be a reasonable cross section of adult students, except in identifiable respects which could be allowed for: its composition was consistent with the little that is known, and the larger amount that is believed, about the adult student body.

The task of finding a control group ideally similar to the adult education sample proved insuperable, but, by kind permission of Sir Douglas Logan, Principal of London University, a small control group was drawn from the staff of Senate House, the University's administrative headquarters. Differences, especially of age, between this group and the sample are discussed in Chapter 3.

## 3. The Sample

Within adult education the only element of choice open was the selection of groups to approach and this turned primarily on the use of lecturers and tutors known personally as likely to be interested and co-operative. Friends among tutors in various parts of England were approached and asked to distribute question papers with a covering letter (Appendix 3), either in their own classes or through other tutors known to them, and to apply what pressure they could to ensure that most of the students returned papers. Paper for answers, graph paper and an envelope for confidential return were supplied.

By this means, papers were obtained from Hampshire, Sussex, the London area, Yorkshire, Durham, Northumberland and Cumberland, and a range of localities from tiny villages to the centres of large cities was included.

The initial hope was to secure reasonably balanced coverage of types of course included in adult education programmes, but, in the end, the sample was over-weighted in favour of physics and mathematics courses. To some extent this was deliberate because the finer details of variation between courses in these subjects was of particular interest. Partly it occurred because friends in other parts of the country tended to be tutors in these subjects and partly it represented the fact (which may be significant) that classes in these subjects produced far better returns than classes in general.

The Senate House Control Group was reached with the help of officers of the Staff Association. They selected the group to be approached and ensured that it was a balanced sample of Senate House staff with respect to staff grades except that the highest grade was excluded because the number of Senior Officers in this grade is very small. Young ladies in their teens and early twenties inevitably formed a much higher proportion of the Control Group than of the main sample.

It is difficult to assess comparisons between Senate House staff and the staffs of large administrative units in general. The whole spectrum of administrative tasks can be found in the University but near-mechanical jobs are probably fewer than in commercial offices, and many of the tasks have an academic flavour. Possibly the University can be more selective in the staff it engages than some organisations.

4. Personal Particulars required from Sample

Participants in the survey were asked to supply potentially relevant personal particulars (on the form reproduced as appendix 4.1). When papers were returned this information was codified and transferred to the score sheet (appendix 4.2).

Item C on this sheet indicated the part of the country concerned (which subsequently appeared not to be significant). Item D distinguished between city centres, suburbs, and rural areas.

For Item F additional information was sought from tutors so that courses could be classified according to subject group, and also according to level - that is according to the previous knowledge explicitly or implicitly required of students in the publicity for the course.

Item K was the age at which formal education ceased: where education had been interrupted by, for example, military service, the figure used was the age at which education of that type would have ended had there been no interruption. Item L was compiled similarly but referred to formal education in mathematics. It proved useful to consider the students 'Forgetting Time' that is, the number of years that had elapsed between the conclusion of mathematics education and the performance of the test: this figure was recorded as Item M.

The five occupation classes employed in many Government surveys was too blunt for the present enquiry from which the "lower" half of the population is excluded. Accordingly, the seven class scale, favoured by sociologists, was employed and entered as Item N.

This scale is as follows:

1. Professional and higher administrative (highly specialised experience plus degree or professional qualifications).

2. Managerial and executive (responsible for initiating and for implementing policy; e.g. personnel manager, head-master of elementary school etc.).

3. Inspection, supervisory and other non-manual, higher grade (with certain authority, e.g. police inspector, teacher.)

4. Inspection, supervisory, and other non-manual, lower grade (with certain responsibility, e.g. costing clerk).

5. Skilled manual and routine grades of non-manual.

6. Semi-skilled manual.

7. Un-skilled manual.

(Reference: Cauter & Downham. The Communication of Ideas. 1954)

Item O on the sheet, Use of Mathematics, was estimated from the occupation quoted with additional guidance from the other information available, e.g. types of courses attended. Three grades were used, namely:

considerable use of mathematics;

some use of mathematics;

minimum use of mathematics.

(Items V and W referred to a hope which had in the end to be abandoned. Tutors were asked to report on the effort and achievement of their students so that any relationship between mathematical ability and personality traits could be investigated. In the event, too few tutors submitted this information for conclusions to be drawn.)

5. Returns

A little over half of the papers were returned by both the adult education sample and the Senate House control group.

In the adult education sample, the number returned appeared to be determined by the attitude of the tutor; returns were consistent with an informal assessment of the energy and interest that particular tutors were likely to exert. Some secured 90%, others only a handful.

By pressing hard over many weeks I eventually secured nearly 100% returns from my own classes and I studied the response to see if I could detect any pattern. It appeared that willingness to return papers reflected character traits rather than ability, although there was a slight tendency for the weak to be reluctant. This might have weighted results where tutors did not urge very strongly because, even in adult students, there is a reluctance to show work which may not please "teacher". I certainly found it difficult to convey to mathematically weak students that I should not be upset or annoyed by slim answer papers and that I wanted a return even if it was small. Other tutors reported similar impressions.

When the papers had been marked those which had arrived in large batches from classes with a good response were compared with similar numbers which had arrived in small batches from classes with a poor response. There appeared to be no significant difference so, in the end, all papers, whether from large or small batches - and even a few papers which arrived singly - were accepted.

A few papers carried unsolicited written comments, generally expressing enjoyment in the performance of what had first appeared as a formidable task. One such - a lady of 57 - wrote: "Comments of Guinea-pig - even if unwanted. I enjoyed doing this but seem to have retained little of the higher mathematics I once achieved. I never understood logarithms." In fact, she appeared to retain a creditable proportion of the mathematics she had left over forty years earlier.

Another lady, a teacher aged 35, wrote: "I attended this course (astronomy) and have enjoyed it only because it was presented with as little of the mathematical side as possible. Sums were always drudgery to me clouding every school day when it happended first thing in the morning. I was more familiar with crosses than with those twirly 'R's and my sense of the rightness of things rebelled at baths with leaky bottoms, useful as such problems might be". Only one elderly lady claimed to have no knowledge of mathematics at all. She had left a private school run by Church of England Sisters at the turn of the century and she firmly indicated: "no instruction in mathematics."

One man, with a good deal of mathematics though certainly not up to the more difficult questions on the paper, had felt bound to work through the entire paper with the help of books. He commented that he had spent every night for a week on the task. This must have been good for him but his paper was excluded from the survey!

## 6. The Composition of the Sample

Table 6.1 gives the composition of sample and control groups with respect to age and sex and comparable figures for the population of England and Wales. Evidently, the adult education sample had a strong male bias, and the control group a strong female bias (row g, cols.1, 2, 11, 12). Both groups show a young bias (cols. 6, 7, 8 ).

These biases in the adult education sample were unexpected for the adult student body is notoriously weighted on the female and elderly sides. The discrepancy appeared to arise from the method of selecting the sample whereby, as discussed in Chapter 3, scientific studies were over-represented by comparison with adult education programmes in general, as shown in Table 6.2.

The effects of this imbalance in the sample were investigated by "normalising" i.e. by "weighting" raw data for age and sex distribution (as in Tables 7.1 and 7.2) to reproduce the subject distribution in adult education as a whole. Tables 6.3 and 6.4 show age and sex distributions for the "raw" sample, the "normalised" sample and for the adult education population.

Clearly the prevalent sex and age imbalance among adult students is related to the subject imbalance in adult education programmes: that is, to the under-representation of "modern" subjects - especially the sciences. If adult education "management" corrected the latter imbalance it would automatically correct the former.

In considering education the 1951 Census Report distinguished young people (under 25) from the rest of the population on the grounds that they reflected the operation of the 1944 Education Act, the development of the welfare state, and the growth of a distinct teenage culture. This distinction has been followed in Table 6.5 which shows the ages at which formal education ended.

The distribution according to sex shows that, bo[illegible] the sample and in the population as a whole, the proportion of women educated beyond the statutory school leaving age is greater than for men. The difference appears to be a little less marked in the sample than in the population but with so small a sample this may not be significant.

The bias in the sample in favour of the better educated is clearly shown. Whereas only 18% of the population has been educated beyond the age of 15, the table shows that only 18% of the sample has not, and that almost all of these were people who completed their education before the war. The proportion of people in adult classes educated to age 16 or above is shown to be about five times the national figure and of people educated to 17 or above it is seven times the national figure for the older group, and nine times the national figures for the younger group.

These points are incidental to the present survey, but they suggest that there is a good adult educational and social case for a deliberate study of them and for explicit recognition of their implications.

Figure 6.6 is a convenient representation of some of the information from Table 6.5 together with information about education in mathematics derived from later tables.

Table 6.7 shows the occurence of higher education (to age 17 or beyond) and of higher mathematics education and also their distributions with respect to occupation group. Extreme caution is necessary in drawing any general conclusions from these figures; for example, in the sample one third of the persons in occupation groups 4, 5, 6 and 7 received higher education and this can hardly be true of the population as a whole!

It is notable that in each occupation group, a smaller proportion of men than women, especially older men, received a higher education. Differences between age groups may represent simply the limited opportunities for higher education before the war: differences between the sexes are a consistent feature of the sample:- women attending adult scientific classes have generally enjoyed a particularly good education. The differences may also reflect social factors - for example lack of higher education used to be a total barrier against the entry of women into higher occupations but the barrier is, perhaps, a little lower today.

Occupation appears to be more strongly correlated with general than with mathematics education. It is a matter of judgement whether this is equivalent to saying that an arts education is most suitable for the higher occupations or that, in the past, the higher occupations have not been accessible to people with a numerate education.

The control group shows that persons whose higher education included mathematics are not very likely to work in university administration. (It would be interesting to examine the administrative staff of a technological concern to see if this still applied.)

The relationships between general education and mathematics education shown in Table 6.8 suggest that the sample is in no way extraordinary. 66% of the sample finished their general and mathematical educations at the same age; 29% continued their education after giving up mathematics; and 5% continued mathematics beyond other subjects. These distributions are shown in Table 6.9 to vary greatly with the age at which formal education ended.

The Table also confirms that mathematics plays (and has played for the 60 - 70 years covered by the survey) a part in almost all education up to what would now be '0' level but, in the population as a whole, the proportion of education without mathematics to ages beyond 16 is certainly greater than in the sample. Figure 6.10, (derived from Tables 7.5 and 6.5) shows that the sample is more heavily weighted on the side of higher education in mathematics than the general body of adult students.

## 7. Adult Educational Aspects of the Sample

The compositions of parts of the sample drawn from different types of adult courses show marked differences. For the purposes of study, courses have been divided into physical sciences, biological sciences and non-sciences, and the physical sciences have been further sub-divided into selective and non-selective, according as publicity for the course implied or did not imply that previous acquaintance was desirable. The effect of the location of courses has also been studied. Location has been taken to mean the place where the course is held - not necessarily where students live - and has been divided into city, suburb and county areas following GOULD, J.D. "The recruitment of adult students" (Vaughan College Papers No.5 (1959)).

The figures in Table 7.1 necessarily reflect the method used to obtain the sample, but the tendency of scientific courses, especially demanding scientific courses, to be concentrated in city centres is consistent with general adult educational statistics.

On the other hand, the apparent concentration of non-scientific courses in county areas is probably an accident of sampling: London tutors in arts subjects proved to be the least co-operative group in approaching or securing replies from their students.

The male bias in city courses is linked with the high proportion of physical science courses in the sample. If the composition of the sample according to subject studies is normalised (as in Chapter 6) the sex ratios become (male : female); city courses 46:54, suburban courses 41:59, county courses 40:60. The first figure is close to that for the London University Extension programme (which contains a preponderance of city courses) namely 47:53. The corresponding figure for the London Tutorial Classes programme where the emphasis is on suburban and county courses is 30:70, but this programme lacks scientific courses and also other types of courses reported by GOULD as attracting a larger male element.

Quantitative comparison with Gould's figures for the distribution of occupation groups is not possible because he used an older occupational classification, traditional in adult education statistics but not in sociology, nevertheless his finding that there were nearly twice as many housewives in suburban and county courses as in city courses is consistent with the present figures.

The distribution of adult students between occupation and age groups in various types of course is shown in Tables 7.2 and 7.3. Table 7.3 shows that the occupation structure is substantially independent of the age structure.

One of his major findings was that members of the higher occupation groups living in suburban or county areas were likely to attend courses in the city centre, but that members of the lower groups were unlikely to do so. He considered that members of the higher occupation groups were not only more likely to work in the city centre and better able to pay fares, but also that they were more disposed to exert themselves to find courses offering demanding treatments or more specialised topics. Columns 7, 8 and 9 of Table 7.2 are consistent with these conclusions.

Three figures relating to the attractions of subjects of study for members of various occupations call for comment. The first - the small proportion of occupation groups 1 and 2 attending biology courses - is considered to be an accident of sampling: the sample for biology was small and many of its members came from a single course for which the descriptive publicity material might have implied (incorrectly) an undemanding treatment of a topic no longer in the news (taxonomy) and might therefore not have attracted the most demanding students; there is no reason to believe that they are <u>necessarily</u> scarce in biology courses. The second notable figure is the comparatively high proportion of people from occupation groups 5, 6 and 7 in non-science courses. Probably this springs at least in part from a combination of the special concern of the W.E.A. to attract these groups with the general scarcity, in W.E.A. programmes, of scientific courses but it may also reflect a reluctance of members of these groups to contemplate scientific studies and perhaps a failure by promoters of science courses to make them palatable at this level.

The third point is that, for occupation groups 1 and 2, the attractiveness of a course appears to depend more on level than on subject; members of these groups are two and a half times more likely to attend selective scientific courses than non-selective courses, either in the sciences or in other subjects.

Figure 7.4 illustrates data from Table 7.2 relating to the age structure of various types of course and includes, for comparison, the structure for the whole country.

This dependence of age structure on the subject of a course is familiar to those adult educationists whose experience covers the necessary range of subjects, though not necessarily to all adult educationists. The comparative scarcity of older students in science courses is not surprising; their formal education was completed during the first three decades of the century when only a minority had much contact with the sciences and the sciences could be treated as a minor factor in society. (In 1900 the number of scientists was about 2% of the number working today). The excess of elderly students in non-science courses (the large majority of adult courses) springs from a like consideration - that most adult education programmes reflect the social and academic values of the formative years of the people concerned. While the scale of the present enquiry does not permit a study of variations of age structure between different sorts of non-science course, it is known that subjects of the present day (such as Transport Studies) are as successful as the sciences in attracting a balanced student body.

The structure of courses with respect to formal education ble 7.5) shows a striking difference between the sexes. If attentio s confined to men, the structures with respect to general education, of classes studying non-selective sciences and non-sciences are substantially the same; and although the proportion of students with higher education naturally increases in selective courses, the change is not dramatic. If attention is turned to women the position changes completely and it becomes nearly true that women without higher education do not join science courses.

For men the structure with respect to mathematics education is similar to that for general education, but for women we come nearly to a position where women with higher mathematics do not join non-science courses! Some difference is to be expected because fewer women than men receive a higher education in mathematics, but whereas the men in non-science courses are reasonably typical of the whole male sample, the women are not typical of the whole female sample. These conclusions are consistent with experience in conducting science courses: one meets a fair number of men with elementary education but very few women.

## 8. The Test

There are plenty of standardized mathematics tests but they have been standardized on children or other special groups by procedures which probably have little relevance for adults and many of them cover a smaller range of mathematical ability than the range to be expected in the present survey. The range might have been covered by using a number of tests but too many questions, especially too many easy questions, might well have irritated the guinea-pigs whose co-operation was essential. Moreover, it was necessary to keep the volume of marking within reasonable bounds. It was also desired to obtain a specific guidance on problems in the popularization of science by discovering what scientifically significant items of mathematics could be expected and what could not.

For all these reasons it was decided to employ a specially constructed test containing the sorts of questions that adults would have faced in their school days. It was neither necessary nor desirable for the test paper to have a modern look. It was decided to include both mechanical exercises and also problems requiring insight into the significance of techniques.

The concept of standardization is, of course, meaningless in any first investigation and it has not been possible to trace any comparable enquiry.

The test was devised as a compromise between the prohibitive amount of labour involved in obtaining <u>all</u> the information that could be envisaged and the impossibility of a repetition to discover anything that had been forgotten. It can be said at once that some reduction in the number of questions should be possible in any future tests.

Questionnaires permitting mechanical marking were considered but it was accepted that, like standardization, they were not practicable when the sorts of responses to be expected were completely unknown.

Forty questions were devised, intended to span mathematics teaching from the end of the junior school syllabus to work done at scholarship level or the first year at the university. None of the questions involved difficult mechanical arithmetic and in all the problems the numbers were chosen to make the calculations easy.

The decision not to go below the 10-year-old level was a matter of judgement based on observation of students in adult classes, and it was justified when there proved to be only a handful of cases in which more detailed probing of junior school mathematics would have been useful. The upper limit was set by the difficulty of investigating higher mathematics by short questions; it would have been necessary to ask for more-or-less protracted mathematical arguments which adults might have been reluctant to undertake and which would have created major difficulties of marking.

The questions were then submitted to experienced teachers: warm thanks must be expressed to Mr. Arthur Worthy, Head Master of Ifield Junior School and to Mr. Norman Arscott, Head of the Mathematics Department at Ifield Grammar School. They were asked to ignore all recent developments in the teaching of mathematics, to consider whether the questions were reasonably typical o. questions which might have been asked some years ago, and to state the ages at which the topics included would have been taught a few years ago.

On Mr. Arscott's advice, three of the more difficult questions were omitted because they might be dealt with at widely divergent ages in different schools. We were unable to arrive at a reasonable method of investigating knowledge of Euclidean geometry (which has traditionally played a considerable part in grammar and comparable schools) and only a single question (which we thought was probably too easy) was included (State the theorem of Pythagoras).

The paper prepared in this way implied a tentative scale of Nominal Mathematical Age; that is, there should be some correlation between the number of questions successfully answered and the age of a grammar school pupil in a school where 'New-Look' syllabuses (Thwaites, Nuffield etc.) were not in use. If we had devised a perfect test we could expect that the ranking order of the questions (according to the frequency with which they were correctly answered) would prove to be the same as the order in which they appeared on the paper. (This turned out to be true only in the most general terms).

It is far from obvious that a test planned in accordance with comparatively recent educational procedures - the last 30 years - has the same significance when applied to people whose education finished sixty years ago. Nor is it certain that the criteria appropriate to a grammar school apply to elementary school education, to prep. school/ public school education, to secretarial colleges, to technical and commercial training establishments, or to a variety of unclassifiable girls' private schools. All that could be foreseen was that the majority of the sample were likely to have been to grammar schools and that no test having the same significance for all people seemed possible even in principle. Consequently, there seemed little point in consulting large numbers of school teachers in the hope of eliminating small differences of emphasis. It was decided to apply the test and to keep an open mind about the significance of the results until the whole batch had been examined.

It will have been noted in appendix 3 that the use of books was permitted but that students were asked to record when they had done this. It would have been impossible to prevent the keener adult students from referring to books and it was considered that, in this way, it would at least be possible to detect when they had done so.

The test paper is appended as appendix 8.

The intention in marking the papers was to give credit for remembrance of method and to pay little attention to the numerical correctness of the answer but it was impossible to apply uniform standards throughout the question paper. A correct numerical answer was generally important in the early questions which were purely arithmetical but it could be given less weight in later questions.

No credit was given if the right result was obtained by inappropriate methods! Many people - often with no great education in mathematics - showed a considerable ability to find numerical answers by intuitive methods, for example in the simple "think-of-a-number" problems(17, 19 and 20). These methods show insight and are greatly to the credit of the people concerned but since this was a survey of knowledge rather than insight, credit was not given.

The large majority of the answer papers were reasonably well set out and understanding of method was reasonably easy to assess. A minority consisted of almost incomprehensible jottings with an answer at the end, and a few were lists of answers only. A firm impression was given, both by written comments and apologies and by a number of indications which could hardly be defined, that many adults retain a juvenile emphasis on 'getting their sums right', and were unable to appreciate that 'teacher' only wanted to know if they remembered the method. About six per cent of the papers returned were rejected because it was impossible to assess their significance.

In order to minimise subjective factors in assessment, the whole job was compressed into a space of two weeks. Only broken time was available so efforts were made to achieve marking sessions of at least two hours with minimum interruption. At the start of each session a few papers were read until one or two awkward points of assessment had been reached. These papers were then put aside; marking commenced and these early papers were brought back towards the end of the session.

Every question was marked out of a maximum of two. This had one use in that it allowed one mark to be awarded where a book had been consulted, but on the whole it was an unnecessary complication reflecting the difficulty of putting aside the habitual attitudes of an examiner (who feels that he must always give what credit he can, even if there is little evidence of complete understanding).

With Question 26 (arc and radius) one mark was allowed for the theoretically valid but unnecessary procedure of using trigonometrical tables to find the sine or tangent of a small angle. This was a fairly frequent occurrence.

In three or four cases a correct result was given for Question 35 (exponential increase) by methods which appeared to be purely arithmetical. In none of them was it possible to detect whether intuition, or some process with which I was not familiar, had been employed, or whether it was a chance possibility arising from the actual numbers in the problem. Fortunately in each case the score was already quite high so one mark was awarded in the knowledge that it made little significant difference to the total score!

It was not difficult to deal with an unforeseen method for "solving" the differential equation in Question 36 in which the two 'd's were cancelled, leading to x=y as the solution!

In Question 3, a surprising number of people did not accept that the brick intended was the same as the brick in Question 2, and in both questions a few people raised the problemof the 'frog' which is a feature of most modern bricks. Mathematics teachers confirm that these difficulties would seldom be raised by children.

## 9. First Results

Nearly ten thousand questions were marked. When papers accompanied by incomplete personal particulars or containing a considerable proportion of answers which could not be assessed had been discarded, 263 were admitted as useable. They contained 5,049 correct answers (in the sense of method correct) out of 9,731 possible answers, that is 51.9%. The distribution of correct answers could be seen immediately to bear some relation to the personal particulars and numbers of correct answers to questions ranged from almost 100% for the simple questions to a very small figure for the most difficult.

Although there was broad agreement between the order of the questions on the paper and the frequency with which they had been correctly answered, there was considerable disagreement in detail. The rank order of questions in terms of numbers of correct answers obtained are given in parenthesis in appendix 8 (the question paper).

The histogram in figure 9.1 shows the distribution of correct answers between the questions. The tendency for questions 10-17 to receive above average numbers of correct answers and for questions 25-32 to receive below average correct answers probably reflects the over-representation in the sample of people educated in mathematics to the level of the middle questions and the under-representation of those who might be expected to handle the later questions. The effects of 'normalization' will be considered in Chapter 10.

The series of histograms in figures 9.2 to 9.9 are similar but are broken down to show differences between the performances of groups (standardized to 100) whose mathematics education finished at 14 or under, fifteen, sixteen etc. to twenty or over.

Fig. 9.10 shows the mean numbers of correct answers obtained in each of these mathematics education groups.

Some questions were consistently answered by more or less people than the method of setting the paper implied: the distribution, for what it is worth, is shown in Table 9.11.

It is a little surprising to note that the first five items in this table (questions better answered than was expected) relate to <u>problems</u> in which the first step must be the translation of a verbal statement into numerical or algebraic form: in general teaching experience, it is this step rather than the mathematical solution which presents difficulty. While these problems were simple, in the sense that no great insight was needed for the translation, there are evidently points here which call for further investigation in any future enquiry. Are the difficulties experienced by children removed by the process of becoming adult? Are the sorts of people who can deal with problems also the sorts who become adult students?

Perhaps success reflects the thorough drilling in problems of this sort that was, and in many cases still is, a feature in most grammar schools. This possibility is supported by the reversal of the pattern in a group of more difficult questions later in the paper, namely 27-30: numbers 27 and 29 called for mechanical operations (in differentiation and integration) while numbers 28 and 30 were problems using these processes, and the pattern of marks 2,0,2,0 was common. This is consistent with experience that drilling in problem-solving techniques is less thorough at the calculus than at the elementary algebra stages, and may also show that, since it occurs at a later age, it is less effective.

It is not easy to account for the high level of success with the graphical questions nos. 31, 33 and 34: it might imply that the visual content of these topics makes them comparatively easy to master and retain for students who think visually.

Question no. 7 on the mechanical subtraction of decimal fractions showed a marked variation of success with level of mathematical education - the best mathematicians producing a surprising number of wrong answers. The impression was received during marking that whereas people with less education in mathematics had set the figures out systematically and performed the operations formally, some of the more able had 'seen the answer in a flash': the figures were intentionally "tricky" and the over-confident fell into the traps they contained.

Success with the complex number problem(no.37) was sometimes baffling: correct answers and working were given on several occasions by people with relatively little mathematics education and no unexpected successes in other questions. Unfortunately, the personal particulars demanded were insufficient to enable a possible link to be studied with training in alternating current technology where complex numbers are employed, but no other explanation can be suggested.

The low level of success achieved with the questions on prime factors and highest common factor (nos. 5 and 6) is interesting. Both topics call for considerable insight into the nature of 'number' and both are notoriously difficult to teach to children so that they are frequently not taught at all in modern practice. They would, however, have been taught in most schools when the 'sample' was at school. In question 5 the technical term 'prime factors' was used but in question 6 the term 'highest common factor' did not appear: this seems not to have affected results. The HCF problem was included because of frequent difficulty, in adult physics courses, in presenting a topic which depends on a HCF argument (Millikan's determination of the electronic charge). In any future enquiry it might be useful to investigate more thoroughly the extent to which these properties of numbers are appreciated.

From the histograms (figures 9.1 to 9.9) relating numbers of correct answers to mathematical education, it can be seen that results are more irregular for the group with least mathematical education than for any other group. It seems likely that this stems from differences between elementary school mathematics syllabuses before the 1944 Education Act and the more modern type of syllabus represented in the present question paper. Questions 1,4,7,10,11,12 and 14, which were correctly answered on a large number of occasions, are purely arithmetical and were certainly taught in the higher classes of pre-war elementary schools. Questions 8,9,13 and 16 which were poorly answered, would have been taught at well below the age of 14 to pupils in grammar schools but would probably not have been taught at all in elementary schools.

## 10. Parameters

If residual mathematical knowledge was to be compared with other factors - age, sex, education etc., it was necessary to express the results of the test in terms of a small number of meaningful parameters but, since the field was virtually uncharted, no attempt was made to define them in advance.

It seemed reasonable to assume that the age at which mathematics education ended was a significant factor and its use in preliminary analysis has been mentioned in Chapter 9. The relationship between what a pupil has been taught and what he knows at the moment when his mathematics education ends is unknown but at the level of these not very difficult questions there is probably no great difference. It was decided to use this as one parameter in the main analysis and it will be referred to hereafter as Mathematics Education Age (MEA).

The test questions were set in a notional order of teaching which was certainly not universal either in time or in different educational establishments. It is possible to imagine a 'perfect' test in which candidates from a perfectly uniform educational system would obtain full marks up to a definite point in the test and no marks thereafter. Nothing as sharp as this could be expected in real life but the results were searched to see if they contained any pattern approximating to it. Since the order in which the questions were set did not correspond to the abilities of many candidates, attempts were made to discover sharper conclusions by re-numbering the questions. Fig. 10.1 shows, for the whole sample, numbers of correct answers for questions re-arranged in rank order: it is as smooth a curve as can reasonably be expected. The histograms in Figures 10.2 to 10.9 show numbers of correct answers for each of the Mathematics Education Age groups with the questions in the same rank order. They should be compared with histograms 9.2 to 9.9 where the questions are in the original order: from the point of view of a social enquirer the second group are distinctly more elegant than the first but there are so many variations in detail - the sizes and positions of peaks and troughs - that the re-arrangement still fails to approximate to the 'perfect' test or to indicate useful parameters.

It seemed possible that the last question correctly answered by a candidate might be significant so this factor was studied both with the questions in the original and in the ranking order. It was evidently correlated with MEA but the scatter was somewhat greater than for total numbers of correct answers and parameters were even harder to discern. A graph was prepared, relating total scores to last questions answered (in rank order), in which different colours were used to denote different MEA groups. It showed that marks would, in general, have been about 10% higher if correct answers had been given up to and including the last question correctly answered, but, once more, a general impression of correlation did not define a useable pattern or suggest suitable parameters.

When these and all other attempts to detect patterns in the raw data had failed it became not only necessary but also justifiable to fall back on total score. This, after all, is implicit in normal examination practice. Viewed with hind-sight, the search for other factors was a waste of time, but it seemed necessary to justify a use of normal examination procedures in an abnormal context with a test of unknown significance. Moreover this protracted examination of the data had the advantage that it would have shown up biases or pre-conceptions built into the test.

The first parameter derived from total scores was Residual Mathematical Age (RMA). Based on the teaching ages assumed to correspond to the questions when the paper was set, scores to be expected at various ages of leaving school or college were calculated and plotted on a graph. A smooth curve was drawn through them (advanced statistical methods were not indicated) and used to define the RMA scale, shown in Table 10.10.

RMA, defined in this way, is proportional to score over most of the range but, because the paper contained few very easy and few very difficult questions, proportionality is lost at the two ends. At the upper end RMA changes so rapidly with score that little reliance was placed on fine details for RMA 17 and above.

The scale turned out to be remarkedly consistent with total numbers of correct answers given by the whole sample. In fig. 10.11 the points represent total numbers of correct answers in rank order (as on histogram 10.1) and the line represents the RMA scale. This correspondence means that assumptions made in defining the scale were consistent with the mathematical performance of the sorts of adults who formed the sample, but direct extension of the concept of RMA to other sorts of adult would need to be made with caution.

A second parameter, Retention, was defined as:

$$\text{Retention} = \frac{\text{RMA}}{\text{MEA}} \times 100\%$$

For the purposes of this enquiry, Retention can probably be verbalized as 'the fraction of his original mathematical knowledge which the candidate retains' but an implication of this verbal statement must be borne in mind if the concept is applied to a wider range, namely, that each year of mathematics education implies an equal increment of mathematical knowledge. This is certainly not a universal principle; some authorities would claim that the acquisition of the concept of number at age 6 or thereabouts is the greatest increment of all; others would regard the final year of an Honours Degree Course as providing the greatest increment.

## 11. Residual Mathematics

It is necessary to begin by saying that the distributions of Residual Mathematical Age and Retention in the sample turn out to be strongly dependent on the type of course attended. The details of this dependence will be considered in Chapter 12; for immediate purposes the normalizing procedure (described in Chapter 6) has been used to produce figures reasonably applicable to the whole body of adult students. It must be stressed that their applicability to adults as a whole can only be a matter of judgement.

The difference between the residual mathematical ability of the two sexes indicated in Table 11.1 appears consistently in all tabulations of the data. Median values of RMA and Retention (that is, values with equal numbers of individuals on each side) are given in Table 11.2.

The absence from the sample of men with low RMA, (consistent for all types of course) is notable (Table 11.1). It can hardly be entirely the result of differences between the mathematics education of men and women, since the proportions of men and women receiving mathematics education to elementary level only (see Table 7.5) were, for the whole sample, 20 and 22% respectively and, for the group attending non-science courses (which showed the greater divergence) 26 and 34%. The much greater difference between numbers of men and women with RMA 10 or under must mean either a greater avoidance of adult education by men with low RMA or else inferior Retention (and perhaps inferior initial understanding) among women. There is no reason to expect the former and some reason to expect the latter since comparatively few women are required by their work to think quantitatively. The difference seems to be real but this enquiry offers no distinctions between innate and environmental causes!

Differences between the sample and control groups also call for comment. Since the women in the control group were predominantly young, and none had passed several years in mathematically undemanding home duties, the smaller proportion in the lowest RMA group is understandable. The proportion of men with low RMA in the control group is much larger than in the sample but it appears from Table 11.3 (RMA and retention versus Occupation) that there is something unusual in the composition of the control group where, in contrast to the much larger sample, higher occupation is independent of residual mathematical ability. (This might be because of the nature of the work or because of the employment policy of the university). It is notable that 20% of the higher occupation parts of the control group exhibit Retention of less than 60 whereas, elsewhere in the survey, the figure never rose above 8%. For the sample, the strong correlation between RMA and occupation is shown in Fig. 11.4.

For the sake of completeness the distribution of RMA and Retention between age groups is given in Table 11.5 but there are no marked conclusions; there is a tendency for the highest values of RMA and Retention to be absent in the highest age group but it is barely significant.

Relationships between residual mathematics and education are shown in detail in Table 11.6 and in simplified diagramatic form in Fig. 11.7. Residual Mathematical Age is predictably correlated with the age at which mathematics education ended (MEA) and only a little less strongly correlated with the age at which a general education ended. This is not surprising since (as discussed in Chapter 6 and shown in Table 6.8) general education and mathematics education are correlated.

Retention is inversely related to education: the more there is to forget the more is forgotten. The diagrams for the control group stress once more that this group has a more marked attachment to only one of the two cultures (the innumerate culture) than the adult education sample.

Table 11.8 and its simplified diagramatic version Fig. 11.9 show how residual mathematics is related to use of mathematics in everyday life (estimated as described in Chapter 4), and to "Forgetting Time" (number of years between the end of mathematics education and the conduct of the test).

The notable thing about "Forgetting Time" is that it is of surprising little significance! 78% of the people whose mathematics education ended nearly 40 years ago still retain more than 70% of their mathematics. The oldest candidate in the sample finished his mathematics education just over 70 years ago and his retention was 87%. Systematic loss of retention with the years is so small as to be masked (in Table 11.8) by fluctuations in detail and the extreme simplification of Fig. 11.9 is needed to demonstrate that it occurs at all. Fig. 11.9 also shows that regular use of mathematics is much more significant than the passage of time: few people, whatever their mathematics education, retain mathematical ability beyond grammar school fourth form level unless they make regular use of the subject.

An important conclusion, perhaps the important conclusion, from the survey is that people retain more mathematics than they say they retain. Thus in adult classes, even among people who had minimum education in mathematics and who use it very little, 74% can be expected to have a knowledge above "11-plus" standard, 94% above third year junior school standard and the majority can be expected to retain basic school algebra and geometry. This could never be guessed from talking to them.

The sample contained too few people who had only recently completed their education for retention in the first two years to be adequately studied but the impression was gained from a scrutiny of the few available examples that most forgetting takes place within one or two years and that retention falls off very slowly thereafter.

## 12. Residual Mathematics and Adult Education

The distribution of residual mathematics in different sorts of adult course is shown in Table 12.1 but its full significance can be seen only against the background of traditional assumptions in adult education namely:

1. Adults join courses to study matters which were neglected in their juvenile education.

2. A traditional range of studies (complacently described as "liberal") forms the ideal programme for interesting the largest and most representative cross-section of the community.

3. Programmes of adult education reflect "demands" prevailing in the community and especially in the most socially purposeful part of the community.

These articles of faith were laid down in the period between 1904 and 1939. They are implicit in most ex cathedra pronouncements about adult education notwithstanding that their validity has been questioned, especially by the lower ranks. The scale of this enquiry was too small to settle questions of validity but it has established that the "articles" cannot be publicly asserted again unless they are vindicated by a bigger and better survey.

Few traditional adult-educationists will be surprised that courses in the physical sciences are attended mainly by people who are reasonably competent in mathematics; they usually regard the physical sciences as something apart from the main stream of adult education in any case.

The absence from non-science courses of people competent in mathematics should be much more startling. It is inconsistent with Article 2 unless we suppose that the mathematically competent are not a significant section of the community. It is also inconsistent with Article 3. Scientific and technical studies (which include mathematics) form a major part of all further and higher education so that the section of the population which provides the main "market" for adult education includes a large number of mathematically competent people; the interests of these people are simply not reflected in adult education programmes. It is true that some have narrow interests confined to their own semi-culture but there are plenty of others.

The present data suggests no explanation for their absence from adult education but there is other evidence that scientists are strongly repelled by tutors and others who purport to speak on social trends, international affairs, philosophy etc., while displaying massive ignorance of interwoven technical or scientific factors. Scientists will, however, readily attend to speakers on these topics who comprehend the sciences even though their expertise in social studies is smaller.

The data also cast doubt on some implicit assumptions about the popularisation of science. It has usually been assumed that the problem is to restate scientific ideas without jargon and without mathematics; these aims have largely been achieved. It is certain that such bogeys were absent from the non-selective physical science courses included in this survey, yet most of the people who joined had more than sufficient mathematics and those who did not possess this mathematical "excess" stayed away. It appears that a universal appreciation of science may depend on the solution of a larger problem (which may not be universally possible in our generation) viz: reduction of the complex set of factors which leads men and women to suppose that they could never appreciate the sciences or would never wish to.

A student's assessment of his or her own RMA seems to be highly significant for choice of subject of study. Moreover fig. 12.2 shows that increments in median RMA for types of class deemed to require mathematics are all greater for women than for men; that is, women do not join courses assumed to be mathematically demanding (the selective courses) unless their mathematical ability is a long way above average.

When the term "numeracy" was coined in the Crowther Report as an analogue of "literacy" - meaning ability to establish reasonable communication with scientists - critics objected that it implied that ability to handle numbers was necessarily involved. They argued that, since a large range of scientific concepts has little or nothing to do with manipulating numbers, the term was misleading. It now appears that the adult education public falls into this error of supposing that science is inextricably bound up with numbers (so that the denotation of numeracy in the restricted sense which Crowther explicitly excluded coincides in practice with its denotation in the broader sense intended). A major task in adult education is to expose this error.

## 13. Implications for Formal and Mathematical Education

The mathematical competence of the sorts of adults covered by the survey is certainly no worse, and probably rather better, than educationists might expect. If comparable enquiries have been made for subjects other than mathematics they have certainly not been traced, but it is hard to imagine that, for example, an investigation into residual French would establish a general retention of school instruction much in excess of the 70 - 80% found here.

We are led, therefore, to consider why, in the particular case of mathematics, there should be this widespread feeling of ineptitude leading to avoidance of numerate thought.

Material presented in the late stages of any course of study is liable to be only partly digested; most of us have probably been conscious of this even while we managed to pass a final examination. If studies are continued into a further stage the process of digestion is completed when work from the early stage is taken for granted and used. But if studies come to an end, this lack of definition in the most advanced work becomes apparent and the inadequate grasp which served for final examination purposes is lost. This process is consistent with the observation (Chapter 11) that most 'forgetting' of mathematics occurs soon after the end of formal instruction.

In mathematics, lack of understanding is stark and demonstrable: most other studies have some inherent "semantic uncertainty". In examinations in other subjects answers may be inadequate but they can seldom be false; if we fail we can put some of the blame on the inscrutable ways of examiners and the stupidity of the questions they ask; if we look for faults in ourselves we need only acknowledge that we had not mastered enough facts or read enough books. In mathematics examinations (at the levels considered here) the 'facts' are before us and the requirements of the examiners are clear; if we fail, it is not only the examiner who concludes that we lack the intellectual equipment - we conclude it ourselves.

Without intruding too far into the field of psychology it is possible to note that some people find that being demonstrably wrong is so uncongenial that they prefer to avoid situations where it might happen. The use of mathematics is one of them.

When this survey had suggested a mechanism of this sort it became a habit to look for it under the conditions of uninhibited discussion found in adult education and three factors have often seemed to co-exist: a lack of mathematical competence, a lack of sympathy for scientific ideas, a preference for leading discussion into the realms of metaphysics or values. This is significant because the criterion for scientific statements is that they contain the possibility of empirical falsification and the characteristic of metaphysical statements is that they do not. Thus it has often seemed that there may be an inherent psychological basis for C.P. Snow's "Two Cultures": some people prefer, or can accept, positions where empirical falsification is possible and some are repelled by them. The latter will be inclined to claim that they do not have "mathematical minds".

It would be interesting and educationally helpful if this dichotomy could be expertly studied before educational reforms intended to bridge the gulf between the two cultures (in either direction) are crystalized!

Meanwhile the survey suggests that current educational pressures move many young people in the direction of innumeracy for whom it is not a psychological necessity, and, since twentieth century society is dependent upon numeracy, this situation is socially unacceptable. Some of the adverse pressures will be relieved as new types of mathematics syllabus come into general use; others implicit in the examination system have yet to be tackled (pointers emerging from the enquiry are discussed below) and change may take a little time. In any case change in juvenile education permeates society very slowly, - too slowly: even after it has been planned it takes many years to be implemented; even after it has been implemented, people educated before the change are dominant in society for 35-40 years and influential for 55-60 years.

The responsibilities of the adult educational agencies, in these circumstances, for the small but socially important minority whom they reach will be considered in the final chapter but it can be said at once that the selective mass media might perform a valuable service by making more demands on the mathematical ability of adults. We have seen that willingness to tackle numerate concepts is positively correlated with use of mathematics. Although consideration of social, economic, strategic, managerial and political affairs is quantitative at policy-making level and although the better commentators must normally be aware of relevant quantitative factors, published commentaries are overwhelmingly qualitative. Have the commentators perhaps been impressed by their public's false assessment of its own mathematical ability? It is suggested that a small but sustained shift of emphasis would not repel the more critical sections of the community; that it would foster better understanding of the subject matter and incidentally encourage a numerate attitude to things in general.

Any suspicion or fear of mathematics that appears during formal education is inevitably reinforced and imprinted by final examinations. However well the pupil or student has digested work from the early stages he is unlikely to be completely master of the work of the final session and the examination - his last formal contact with mathematics - requires him to regurgitate or manipulate ideas of which he has no firm grasp and of which, in mathematics more than in any other subject, he knows he has no firm grasp. We have already said that, if he proceeds to the next educational stage he soon achieves this grasp but (for something like 90% of candidates) any final examination is the last examination of all. They are left with a consciousness of inadequacy which affects them deeply and permanently.

Since teachers in old-style grammar schools and in colleges are, or were until recently, obsessed by the minority who will proceed to the next stage they are unlikely to initiate change for any social or educational reasons. Moreover, since they assess their own competence by the results of final examinations, we have a circular situation unlikely to be challenged from within.

It appears that an external assessment more like a "product evaluation" and less like a "final factory inspection" would be valuable. Residual understanding some time after the end of the course and after the compulsory examination would be the socially valuable criterion, and if it were investigated it might well call into question the final years of many courses. If the present survey could be accepted as a pilot for such an enquiry - even in the single field of mathematics - it would have served a useful purpose and there seems no reason why comparable pilot enquiries, leading to larger investigations in other fields of study, should not be valuable.

Since it seems that, at least in mathematics, one or two years is sufficient for most "forgetting", this larger enquiry should not prove difficult. It is probable that many schools and colleges could assist an external agency - say a Department of Education - to secure the co-operation of a reasonable number of their recent pupils and students (within, say, two years of leaving) in undertaking a modest test in subjects other than those used in work or further study. If the test included (like the present test) questions covering all stages of instruction, and if it was backed (as this one was not) by a questionnaire on attitudes, many of the present comments - credible but unproven - could be probed and desirable modifications to present curricula indicated.

## 14. Implications for Adult Education

This enquiry was initially undertaken for purposes in adult education and these final comments relate to adult education.

While it is hardly surprising that the types of adult found in adult courses are strongly correlated with the types of courses offered, it is surprising that some adult educationalists still bewail the unrepresentative character of the student body as if it reflected something other than their own actions.

It is generally accepted today (and this enquiry amply confirms) that, apart from an exceptional and probably dwindling minority, adult education appeals mainly to people who had at least a reasonably good juvenile education. Before the Second War numeracy was a minority concern in education and a negligible concern in female education, whereas since the War all secondary education, especially male education, has included a considerable amount of numerate education while sixth form studies and further education have been predominantly scientific and technical. (This assertion tends to be questioned in circles where little attention is paid to the large volume of education concerned with City and Guilds, National Certificates, Dip.Tech. and comparable courses). It follows that massive innumeracy is a natural characteristic of the elderly and the female and it also follows, as surely as night follows day, that a massively innumerate programme of adult education will attract the elderly and the female. (This is the feature so often deplored). It is evident that a programme which bore some sort of resemblance to educational trends since the Second War could attract a more representative sample of the population.

Although society stands in great need of the contribution which the adult education agencies might make to a more general appreciation of numerate thought, the adult education agencies are unlikely to achieve this sort of balance of programme within the next fifteen years. A pre-requisite for a balanced programme is a balanced staff (including the higher grades). The present staff is overwhelmingly innumerate and, with present resources and trends, the recruitment necessary to achieve a balance must take a hundred years!

Meanwhile, this enquiry has enabled limited scientific resources in one adult educational agency (London University Extension) to be deployed more effectively, and it is hoped that it will be similarly useful to others.

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# APPENDICES
# TABLES &
# DIAGRAMS

NUMBERED TO CORRESPOND WITH CHAPTERS IN THE TEXT

Appendix 3

## E X P L A N A T O R Y   L E T T E R

UNIVERSITY OF LONDON

Department of Extra-Mural Studies

M A T H E M A T I C S   S U R V E Y

We should very much appreciate your co-operation in this survey. It is intended to tell us something about the mathematics which adults are still able to use some years after leaving school.

You are asked to answer as many of the questions on the attached sheet as you can, taking as long as you like over it. The questions cover a wide range of mathematical ability and only if you have done University mathematics (or the equivalent) will you be able to answer them all; you may not be able to tackle any (in that case we still want to know about it).

You are at liberty to refer to books to jog your memory if you wish but if you do so please put a note to that effect on your answer to the question (s) concerned.

Obviously, the nature of your schooling, the time that has elapsed since, the extent to which you have used mathematics in your job, etc. will affect your performance. So that we can allow for these things please fill in the "Personal Particulars".

I am very conscious of the demand this makes on you but I hope you will feel able to help because a survey of this type can give misleading results if what sociologists call the "no-contact" rate is too high. May I stress that you should certainly not fail to answer the questions just because you think your mathematics is "shaky"; that is what we want to know!

Thank you!

H.G. FROST
Staff Lecturer

Senate House, London, W.C.1.

Appendix 4.1

PERSONAL PARTICULARS FORM

UNIVERSITY OF LONDON

Department of Extra-Mural Studies

MATHEMATICS SURVEY

(Personal Particulars)

1. Course(s) being attended now..................................
.................................................................
.................................................................

2. Age........................ Sex...........................

3. Types of school, college, etc. attended (Names not necessary)

1.................... up to age...........................

2.................... up to age...........................

3.................... up to age...........................

4.................... up to age...........................

4. Age at which instruction in mathematics ceased................

5. Examinations passed and/or qualifications in MATHEMATICS

.................................................................

.................................................................

6. Examinations passed and/or qualifications in ALL SUBJECTS

.................................................................

.................................................................

7. Occupation (work actually done)...............................

---

NAME.............................................................

ADDRESS..........................................................

.........................................................

Appendix 4.2

## SCORE SHEET: AS USED IN ANALYSIS OF RESULTS

### MATHEMATICS SURVEY

SCORE: 1.............
2.............
3.............
4.............
5.............
6.............
7.............
8.............
9.............
10.............
11.............
12.............
13.............
14.............
15.............
16.............
17.............
18.............
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25.............
26.............
27.............
28.............
29.............
30.............
31.............
32.............
33.............
34.............
35.............
36.............
37.............
T.............

A. Serial No..................A
B. . . . . . . . . .............B
C. District . . . ..............C
D. Locality . . . ..............D
E. . . . . . . . . .............E
F. Subject & Level
or other code . .............F
G. . . . . . . . . .............G
H. . . . . . . . . .............H
I. Sex . . . . . . .............I
J. Age.. . . . . . .............J
K. Education to age............K
L. Maths. to age . .............L
M. . . . . . . . . .............M
N. Occupation class............N
O. Use of maths. . .............O
P. . . . . . . . . .............P
Q. Score . . . . . .............Q
R. Last question . .............R
S. Ditto, re-numbered..........S
T. . . . . . . . . .............T
U. . . . . . . . . .............U
V. Assessed effort .............V
W. Assessed achievement.........W
X. . . . . . . . . .............X
Y. . . . . . . . . .............Y
Z. . . . . . . . . .............Z

## Table 6.1

### SEX & AGE DISTRIBUTION: for Sample & Control Groups, Comparative figures for England & Wales.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SAMPLE | | | | | | England & Wales (see Note 1) | CONTROL | | | | | |
| | NUMBERS | | | PER CENT | | | | PER CENT | | | NUMBERS | | |
| **Sex:** | M | F | ALL | M | F | ALL | ALL | ALL | M | F | ALL | M | F |
| Age Group | | | | | | | | | | | | | |
| (a) 18-24 | 18 | 22 | 40 | 11 | 21 | 15 | 13 | 37 | 0 | 53 | 10 | 0 | 10 |
| (b) 25-34 | 38 | 23 | 61 | 24 | 22 | 23 | 21 | 26 | 12 | 32 | 7 | 1 | 6 |
| (c) 35-44 | 39 | 24 | 63 | 25 | 23 | 24 | 23 | 15 | 38 | 5 | 4 | 3 | 1 |
| (d) 45-54 | 34 | 16 | 50 | 22 | 15 | 19 | 20 | 11 | 25 | 5 | 3 | 2 | 1 |
| (e) 55-64 | 22 | 12 | 34 | 13 | 12 | 13 | 15 | 11 | 25 | 5 | 3 | 2 | 1 |
| (f) Over 64 | 8 | 7 | 15 | 5 | 7 | 6 | 8 | 0 | 0 | 0 | 0 | 0 | 0 |
| (g) Total | 159 | 104 | 263 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 27 | 8 | 19 |

**Notes:**

1. The percentages shown are based on the 1951 Census but are modified to reflect the absence from the sample and control group of people under 18 and the small numbers over 65. 79% of the sample falls between the ages of 25 and 64 compared with 54% of the national population. The national figures (column 7) within that age range have therefore been multiplied by a factor 79/54 and figures for the groups under 25 and over 64 have been corrected by simple proportion.

2. In the 1961 Census Report the sex ratio (number of females per 1,000 males) is given as 1,066 for the whole population; 1,083 for conurbations; 1,015 for rural areas.

## Table 6.2

### SUBJECTS STUDIED IN ADULT COURSES: for the sample & for Extra-Mural Courses as a whole

(Rows shown as 100%)

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | SUBJECT OF ADULT COURSE | | | |
| | PHYSICAL SCIENCE | BIOLOGICAL SCIENCE | NON-SCIENCE | |
| (a) SAMPLE | 61 | 10 | 29 | 100% = 252 PERSONS |
| (b) EXTRA-MURAL COURSES[1] | 5 | 6 | 89 | 100% = 5248 COURSES |

Note:

1 From report of Universities Council for Adult Education

## Table 6.3

### AGE DISTRIBUTION in Sample, Normalised Sample & Adult Education 'population'

(Columns shown as 100%)

| | (1) RAW SAMPLE | (2) NORMALISED SAMPLE | (3) ADULT EDUCATION POPULATION |
|---|---|---|---|
| AGE: | | | |
| (a) 34 and Under | 38 | 34 | 34 |
| (b) 35 - 54 | 43 | 37 | 43 |
| (c) 55 and Over | 19 | 29 | 23 |

100% = 263 Persons

## Table 6.4

### SEX DISTRIBUTION in Sample, Normalised Sample & Adult Education 'population'

(Columns shown as 100%)

| | | (1) RAW SAMPLE | (2) NORMALISED SAMPLE | (3) ADULT EDUCATION POPULATION. |
|---|---|---|---|---|
| (a) | MALES | 60 | 42 | 49 |
| (b) | FEMALES | 40 | 58 | 51 |

100% = 263 Persons

## Table 6.5

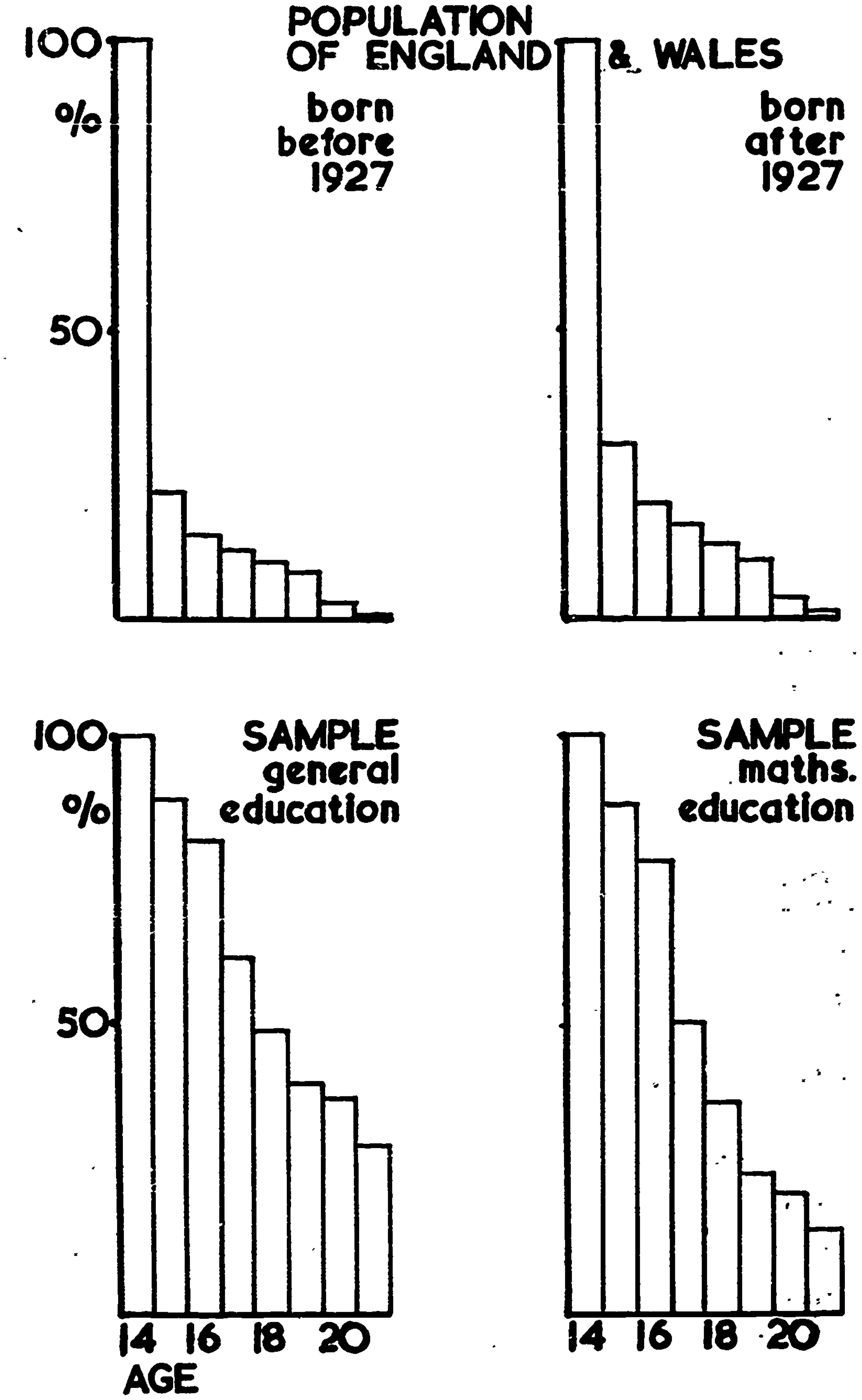
AGE AT WHICH FORMAL EDUCATION ENDED vs. AGE & SEX: for Sample & for England & Wales.

(Columns shown as 100%)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | MALES | | | | FEMALES | | | | SAMPLE M & F | | |
| AGE AT END OF EDUCATION: | 24 & UNDER | | 25 & OVER | | 24 & UNDER | | 25 & OVER | | 24 & UNDER | 25 & OVER | ALL AGES |
| | E&W | SAMPLE | SAMPLE | E&W | E&W | SAMPLE | SAMPLE | E&W | | | |
| (a) 15&UNDER | 81 | 0 | 20 | 87 | 78 | 5 | 20 | 83 | 3 | 20 | 18 |
| (b) 16 | 11 | 12 | 26 | 7 | 11 | 5 | 18 | 8 | 8 | 23 | 21 |
| (c) 17&OVER | 8 | 88 | 54 | 7 | 11 | 91 | 62 | 9 | 89 | 57 | 61 |
| (d) 100% | | 16 | 143 | | | 22 | 82 | | 38 | 225 | 263 |

Fig. 6.6

AGE TO WHICH EDUCATION CONTINUED



# Table 6.7

## GENERAL & MATHEMATICS EDUCATION vs. SEX, AGE & OCCUPATION GROUPS: for Sample & Control Group.

(Columns 1, 2, 4, 5, 7, 8, 10, 11 and rows a, b, c, d, f, g show percentages of persons in the sex/age/occupation groups concerned whose education continued to age 17 or beyond.

The remaining rows and columns show total numbers of persons regardless of the age at which education ended.)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| OCCUPATION GROUP → | 1 & 2 | | NO. IN GROUP | 3 | | NO. IN GROUP | 4,5,6,7 | | NO. IN GROUP | ALL | | TOTAL |
| | UNDER | OVER | | UNDER | OVER | | UNDER | OVER | | UNDER | OVER | |
| AGE GROUP → | 45 | 44 | | 45 | 44 | | 45 | 44 | | 45 | 44 | |
| **SAMPLE** | | | | | | | | | | | | |
| GENERAL EDUCATION 17 & OVER | | | | | | | | | | | | |
| (a) M. | 100 | 81 | 43 | 69 | 46 | 57 | 34 | 16 | 57 | 65 | 44 | 157 |
| (b) F. | 100 | 100 | 12 | 89 | 93 | 43 | 33 | 43 | 48 | 69 | 69 | 103 |
| MATHS EDUCATION 17 & OVER | | | | | | | | | | | | |
| (c) M. | 69 | 60 | 43 | 59 | 50 | 57 | 39 | 20 | 57 | 56 | 42 | 157 |
| (d) F. | 68 | 100 | 12 | 43 | 80 | 43 | 28 | 35 | 48 | 43 | 58 | 103 |
| (e) NO. IN GROUP | 36 | 19 | 55 | 57 | 43 | 100 | 67 | 38 | 105 | 160 | 100 | 260 |
| **CONTROL** | ALL AGES | | | ALL AGES | | | ALL AGES | | | ALL AGES | | |
| (f) GENERAL EDUCATION 17 & OVER | 66 | | | 34 | | | 50 | | | 48 | | |
| (g) MATHS EDUCATION 17 & OVER | 0 | | | 17 | | | 22 | | | 19 | | |
| (h) NO. IN GROUP | 3 | | | 6 | | | 18 | | | 27 | | |

# Table 6.8

## DISTRIBUTION OF GENERAL EDUCATION, MATHEMATICS EDUCATION & SEX: for Sample.

| AGE AT WHICH MATHS EDUCATION ENDED: ↓ | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AGE AT WHICH FORMAL EDUCATION ENDED | | | | | | | | | | % | | Nos. | |
| | Under 15 | 15 | 16 | 17 | 18 | 19 | 20 | Over 20 | TOTAL | % | M. | F. | M. | F. |
| (a) UNDER 15: | 28 | 0 | 3 | 0 | 1 | 0 | 0 | 1 | 33 | 13 | 11 | 14 | 18 | 15 |
| (b) 15: | 0 | 13 | 0 | 3 | 0 | 0 | 3 | 4 | 23 | 9 | 9 | 9 | 14 | 9 |
| (c) 16: | 1 | 0 | 47 | 3 | 4 | 1 | 6 | 14 | 76 | 29 | 30 | 28 | 47 | 29 |
| (d) 17: | 0 | 1 | 0 | 22 | 3 | 2 | 1 | 8 | 37 | 14 | 11 | 8 | 18 | 19 |
| (e) 18: | 0 | 0 | 0 | 0 | 18 | 1 | 2 | 10 | 31 | 12 | 11 | 13 | 18 | 13 |
| (f) 19: | 1 | 2 | 1 | 0 | 0 | 3 | 0 | 2 | 9 | 3 | 4 | 2 | 7 | 2 |
| (g) 20: | 0 | 0 | 2 | 0 | 0 | 0 | 9 | 4 | 15 | 6 | 6 | 6 | 9 | 6 |
| (h) OVER 20: | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 33 | 39 | 15 | 17 | 11 | 28 | 11 |
| (i) TOTAL NO. | 30 | 18 | 54 | 30 | 26 | 8 | 21 | 76 | 263 | 100 | 100 | 100 | 159 | 104 |
| (j) TOTAL % | 11 | 7 | 21 | 11 | 10 | 3 | 8 | 29 | 100 | | | | 60 | 40 |
| (k) % MALE | 11 | 9 | 24 | 11 | 10 | 3 | 5 | 28 | 100 | | | | | |
| (l) % FEMALE | 12 | 4 | 25 | 12 | 10 | 4 | 12 | 31 | 100 | | | | | |
| (m) NO. MALE | 17 | 14 | 38 | 18 | 16 | 4 | 8 | 44 | 159 | 60 | | | | |
| (n) No. FEMALE | 13 | 4 | 26 | 12 | 10 | 4 | 13 | 32 | 104 | 40 | | | | |

Note:

Of 263 total: 173 lie on the main diagonal (general & maths education finished together)

76 lie above it (general education continued beyond maths education)

14 lie below it (maths education continued after general education).

Several of these 14 followed advanced technical courses in the Services as (e.g.) war time radar staff.

## Table 6.9

# RELATIONSHIP BETWEEN GENERAL EDUCATION & MATHEMATICS EDUCATION.

(Education groups shown as 100%; sizes of groups in row (d) ).

| | | (1) | (2) | (3) | (4) |
|---|---|---|---|---|---|
| | | GENERAL EDUCATION TO AGE: | | | |
| | | 15 & UNDER | 16 | 17 & 18 | 19 & OVER |
| MATHS. EDUCATION CEASED | | | | | |
| (a) | BEFORE | 0 | 6 | 25 | 56 |
| (b) | WITH | 85 | 87 | 71 | 43 |
| (c) | AFTER | 15 | 7 | 4 | 1 |
| GENERAL EDUCATION | | | | | |
| (d) | 100% | 48 | 54 | 56 | 105 |

Fig. 6.10

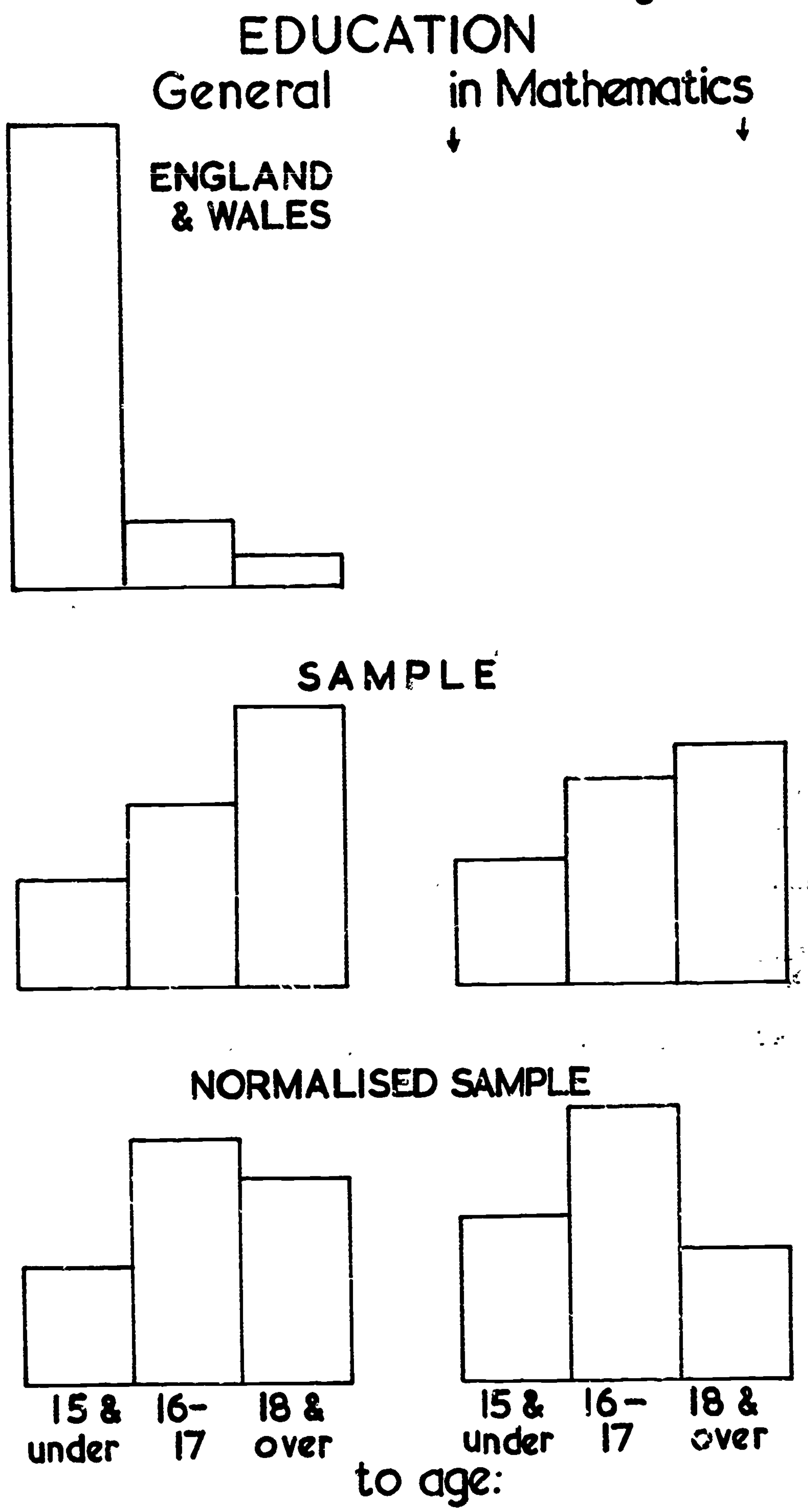

EDUCATION
General
in Mathematics
ENGLAND & WALES
SAMPLE
NORMALISED SAMPLE
15 & under
16-17
18 & over
15 & under
16-17
18 & over
to age:

Table 7.1

SUBJECT & LOCATION OF ADULT COURSES: SEX DISTRIBUTION.

(Percentages except column 6 and row d which show actual numbers)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| | SUBJECTS | | | | | | |
| | PHYSICAL SCIENCE selective | non-selective | BIOL. SCIENCE | NON-SCIENCE | ALL | TOTAL NUMBERS | SEX RATIO M/F |
| LOCATION | | | | | | | |
| (a) CITY | 97 | 65 | 46 | 26 | 61 | 152 | 67/33 |
| (b) SUBURB | 3 | 21 | 37 | 22 | 18 | 46 | 56/44 |
| (c) COUNTY | 0 | 14 | 17 | 52 | 21 | 54 | 44/56 |
| (d) NUMBERS: 100% | 65 | 91 | 24 | 72 | 252 | 252 | |
| (e) SEX RATIO | 80/20 | 66/34 | 50/50 | 39/61 | 60/40 | | |

# Table 7.2

## DISTRIBUTION OF OCCUPATION & AGE GROUPS with respect to SUBJECTS & LOCATIONS of Adult Courses.

(Subject and location groups shown as 100%. Actual numbers in column 6 and row e)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| | SUBJECTS | | | | | | LOCATIONS | | |
| | PHYSICAL SCIENCE | | | | | NUMBERS | CITY | SUBURB | COUNTY |
| | Selective | Non-Selective | BIOL. | NON-SCIENCE | ALL | | | | |
| **OCCUPATION GROUPS** | | | | | | | | | |
| (a) 1 & 2 | 41 | 16 | 4 | 14 | 21 | 52 | 24 | 19 | 11 |
| (b) 3 | 32 | 41 | 59 | 32 | 38 | 95 | 36 | 37 | 43 |
| (c) 4 | 18 | 34 | 29 | 29 | 28 | 71 | 30 | 24 | 28 |
| (d) 5,6, 7 | 9 | 9 | 8 | 25 | 13 | 34 | 10 | 20 | 18 |
| (e) 100% | 65 | 91 | 24 | 72 | 252 | 252 | 152 | 46 | 54 |
| **AGE GROUPS** | | | | | | | | | |
| (f) UNDER 35 | 53 | 35 | 12 | 34 | 37 | 94 | | | |
| (g) 35 - 54 | 35 | 49 | 67 | 35 | 44 | 109 | | | |
| (h) OVER 54 | 12 | 15 | 21 | 31 | 19 | 49 | | | |

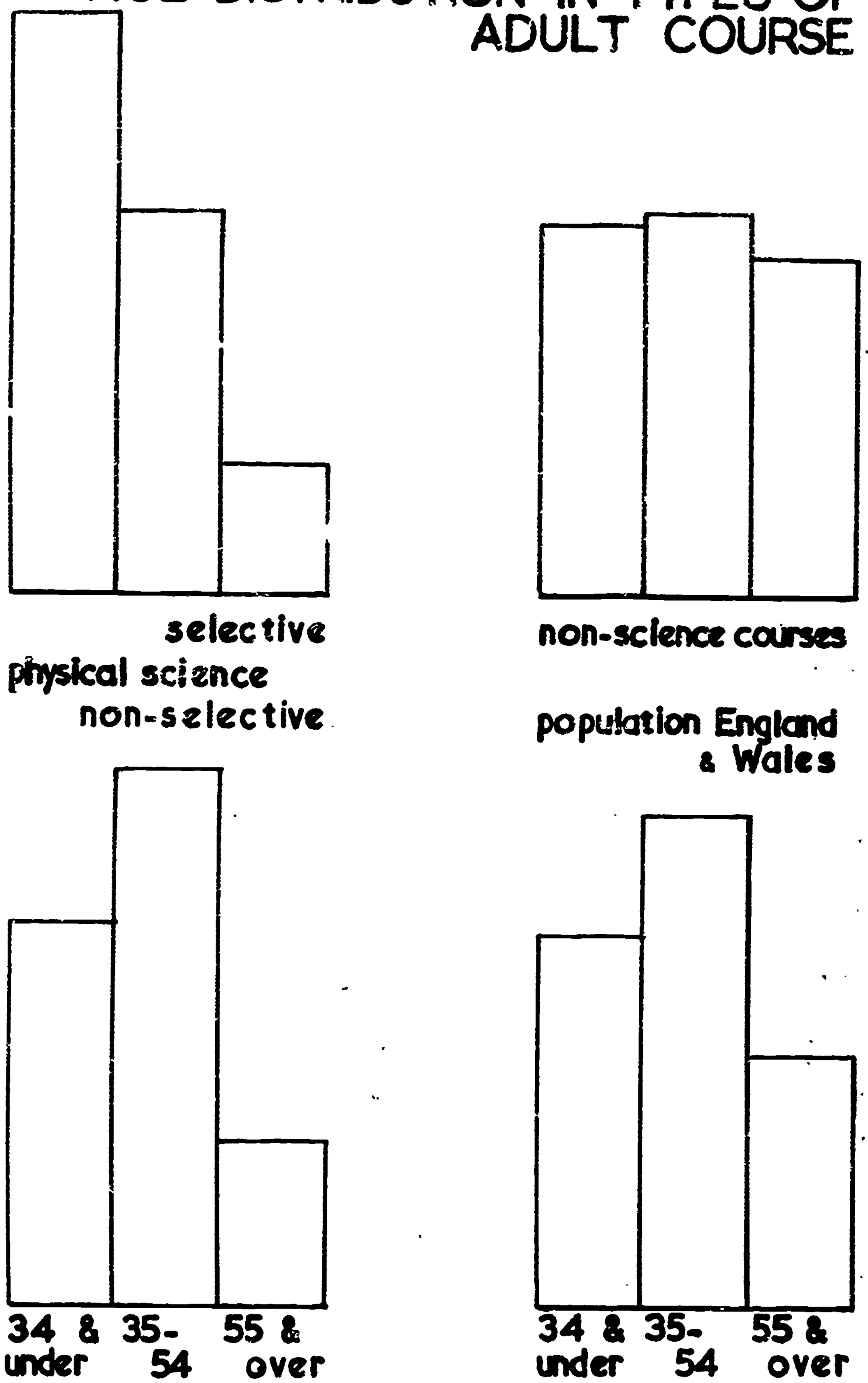
# Table 7·3

## OCCUPATION vs. AGE GROUP in the Sample.

(Age groups show . 100% with sizes of groups in row e)

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | AGE GROUPS | | | |
| | UNDER 35 | 35-44 | OVER 44 | ALL |
| OCCUPATION GROUPS: | | | | |
| (a) 1 & 2 | 20 | 23 | 18 | 20 |
| (b) 3 | 41 | 33 | 44 | 39 |
| (c) 4 | 25 | 34 | 25 | 28 |
| (d) 5, 6 & 7 | 14 | 10 | 14 | 13 |
| (e) 100 % | 98 | 61 | 99 | 258 |

Fig. 7.4
AGE DISTRIBUTION IN TYPES OF ADULT COURSE



## Table 7.5

## EDUCATION & MATHS. EDUCATION vs. SUBJECT & LOCATION OF ADULT COURSE & Sex.

(Subject and location groups shown as 100%. Actual numbers in columns 6 and rows d, h, and 1)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| | SUBJECTS | | | | | | LOCATIONS | | |
| | PHYSICAL SCIENCE | | | | | | | | |
| | Selective | Non-selective | BIOL. | NON-SCIENCE | ALL | NUMBERS | CITY | SUBURB | COUNTY |
| **EDUCATION TO AGE:** | | | | | | | | | |
| **Males** | | | | | | | | | |
| (a) UNDER 16 | 24 | 18 | 24 | 18 | 20 | 31 | | | |
| (b) 16 - 17 | 25 | 43 | 43 | 43 | 37 | 55 | | | |
| (c) OVER 17 | 51 | 39 | 33 | 39 | 43 | 65 | | | |
| (d) 100% | 51 | 60 | 12 | 28 | 151 | 151 | | | |
| **Females** | | | | | | | | | |
| (e) UNDER 16 | 9 | 9 | 16 | 24 | 17 | 17 | | | |
| (f) 16 - 17 | 5 | 14 | 51 | 33 | 25 | 25 | | | |
| (g) OVER 17 | 86 | 77 | 33 | 43 | 58 | 59 | | | |
| (h) 100% | 14 | 31 | 12 | 44 | 101 | 101 | | | |
| **Both** | | | | | | | | | |
| (i) UNDER 16 | 21 | 15 | 22 | 22 | 19 | 48 | 18 | 20 | 21 |
| (j) 16 - 17 | 21 | 34 | 45 | 36 | 32 | 80 | 34 | 34 | 27 |
| (k) OVER 17 | 58 | 51 | 33 | 42 | 49 | 124 | 48 | 46 | 52 |
| (l) 100% | 65 | 91 | 24 | 72 | 252 | 252 | 152 | 46 | 54 |
| **MATHS. EDUCATION TO AGE:** | | | | | | | | | |
| **Males** | | | | | | | | | |
| (m) UNDER 16 | 18 | 20 | 16 | 26 | 20 | 30 | | | |
| (n) 16 - 17 | 25 | 51 | 51 | 42 | 41 | 62 | | | |
| (o) OVER 17 | 57 | 29 | 33 | 32 | 39 | 59 | | | |
| **Females** | | | | | | | | | |
| (p) UNDER 16 | 9 | 9 | 24 | 34 | 22 | 22 | | | |
| (q) 16 - 17 | 27 | 46 | 68 | 50 | 47 | 47 | | | |
| (r) OVER 17 | 64 | 45 | 8 | 16 | 31 | 32 | | | |
| **Both** | | | | | | | | | |
| (s) UNDER 16 | 18 | 16 | 20 | 30 | 21 | 53 | 20 | 25 | 21 |
| (t) 16 - 17 | 24 | 49 | 59 | 48 | 43 | 108 | 39 | 50 | 46 |
| (u) OVER 17 | 58 | 35 | 21 | 22 | 36 | 91 | 41 | 25 | 33 |

Appendix 8

## QUESTION PAPER

UNIVERSITY OF LONDON

Department of Extra-Mural Studies

MATHEMATICS SURVEY

1. $12765 \div 37$ (1)
2. What is the volume of a standard brick, 9" x $4\frac{1}{2}$" x 3" ? (2)
3. What is the surface area of a standard brick? (9)
4. $4\frac{1}{4} - 1\frac{3}{5} - \frac{9}{10}$ (4)
5. Express 3432 in prime factors. (16)
6. 5 paper bags, each containing an unknown number of identical marbles are found to weigh 70, 126, 154, 224, 266 grams. What is the greatest possible weight of a single marble? (17)
7. $1.01 - 0.1 - 0.001 + 10$ (5)
8. Solve:- $z - \frac{2z}{7} = 10$ (8)
9. State the Theorem of Pythagoras. (10)
10. If six men can build a scientific instrument in 35 working days how long would five men take to build three such instruments? (7)
11. A, B and C buy a business: A pays $\frac{2}{5}$ of the cost, B pays $\frac{7}{15}$ of the cost and C pays the remainder viz: £966. What did the business cost? (3)
12. If 1 inch = 2.54 centimetres how many inches are equivalent to 63.5 cm.? (6)
13. What is the area of a circular plot of ground enclosed by a fence 100 yards long? (18)
14. How long is the side of a square whose area is 10816? (11)
15. A Spiral spring is suspended from one end and its length is measured when different weights are attached to the other end:- (14)

| | | | | | |
|---|---|---|---|---|---|
| Weight in grams | 10 | 15 | 30 | 50 | 75 |
| Length in cms. | 22 | 24 | 30 | 38 | 48 |

Draw a graph to show the relation between the length and the load and use it to find the load if the length is 42 cm.

16. If $y = -\frac{1}{2}$ and $z = -\frac{1}{3}$ find the value of $\frac{1}{y} - \frac{1}{z}$ (19)

17. Find a number such that if you add 7 and divide the sum by 5 you will get the same answer as if you had subtracted 1 and then divided by 3. (12)

18. The length of a line is 3.2 inches. Measurement by means of a faulty instrument gives the value 3.4 inches. Find the error per cent. (13)

19. Find two numbers whose sum is 90 and such that one third of the smaller is equal to one seventh of the larger. (14)

20. I think of a number, then square it and add the original; the result is 56. What is the number? (22)

21. P is a point on the side AB of triangle ABC such that AP = PC = CB. If CP bisects the angle ACB, calculate angle BAC. (24)

22. The shadow of a tree was found to be 80 feet long when the angle of elevation of the sun was $36^\circ 52'$. Find the height of the tree (cos. $36^\circ 52' = 0.8$). (23)

23. Simplify:- $b^{\frac{3}{4}} \times b^{\frac{1}{4}}$ (20)

24. Simplify:- $n \times n^{1\frac{1}{4}} \div n^{3\frac{3}{4}}$ and express the result in a form that does not involve indices. (21)

25. If log y + log z = 3 give the value of y.z. (logs to base 10) (25)

26. Find the diameter of the Moon if it subtends an angle of 31' at a point on the earth at a distance of 240,000 miles. (32)

27. Give the differential coefficient of $x^{30}$. (26)

28. A ladder 34 feet long rests in a vertical plane with one end on a horizontal road and the other against a vertical wall. If the lower end is pulled away from the wall with a velocity of 10 feet per minute find the rate at which the (33)

upper end is descending at the instant when the foot of the ladder is 16 feet from the wall.

29. Write down the integral of $3x^2 - 2x + 1$ (31)

30. An elastic string of natural length 6 feet could be stretched to twice its length by a force of 4lb. wt. Find the work done in stretching it from 7 ft. length to 8 ft. (35)

31. If a straight line graph has slope 3 and intercept -7 give the value of y when x = 3. (27)

32. A boat with maximum speed 10 knots wishes to proceed due N while a 5 knot tide is running E - W. In what direction must the boat head and what will be its resulting velocity towards its objective? (29)

33. Draw the graph of $4x^2 + 9y^2 = 36$. (28)

34. What is the name given to the point $(\sqrt{5},0)$ on the above graph? (30)

35. The rate at which the height of water falls in a tank with a leaky bottom is proportional to the height of water in it. Initially the height is 10 feet and the level falls at the rate of $\frac{1}{2}$ inch per minute. After how long will the level have fallen to 1 foot? ( )

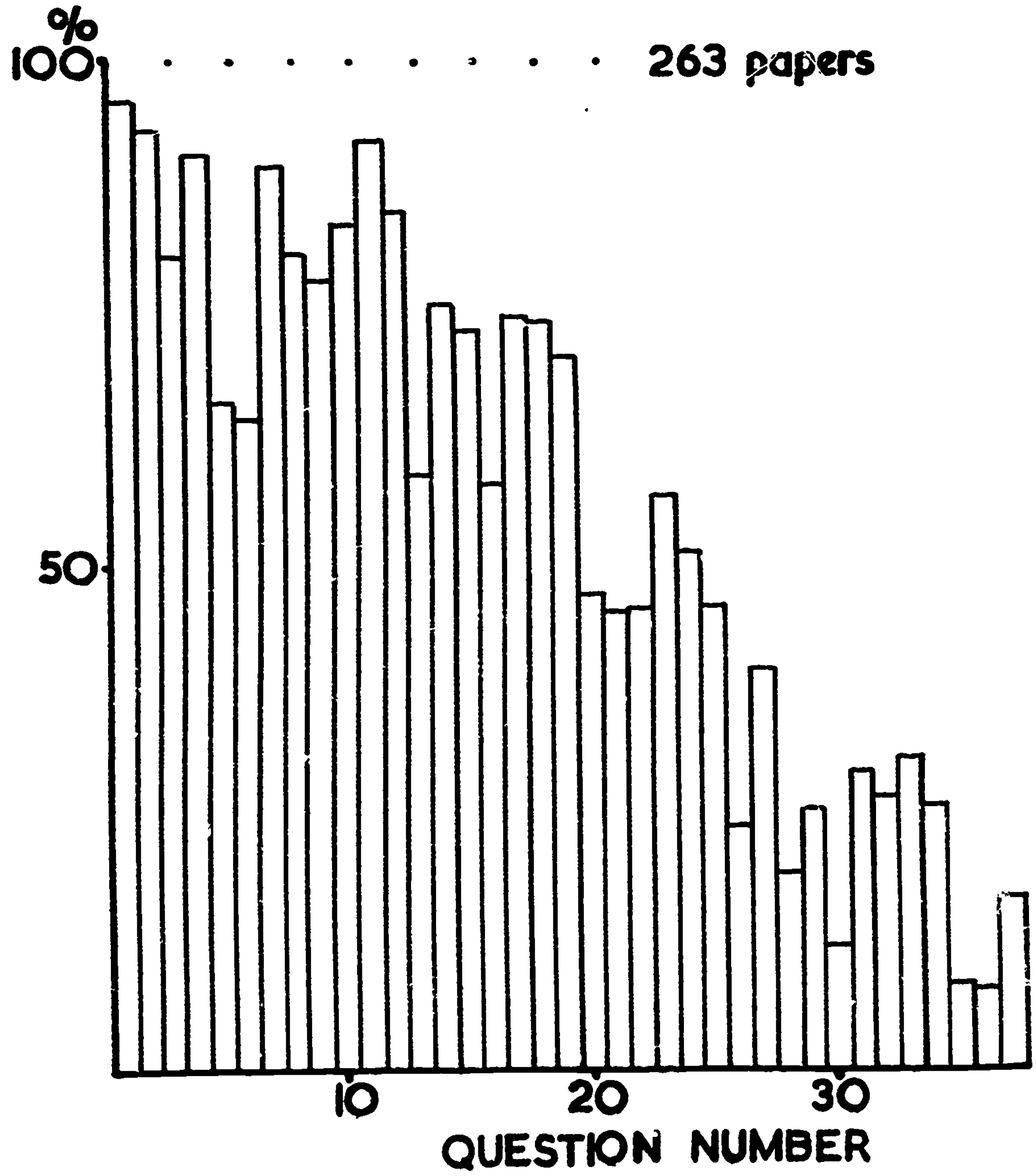
36. Solve:- $\frac{dy}{dx} = \frac{x^2 + y^2}{2x^2}$ (37)

37. If $a = \cos b + i.\sin b$ express $a + \frac{1}{a}$ in terms of b. $(i = \sqrt{-1})$ (34)

The figures in parentheses after the questions are the positions of the questions in rank order according to the number of times they were correctly answered. (See Chapter 9)

Fig. 9.1

CORRECT ANSWERS TO QUESTIONS



# CORRECT ANSWERS BY M.E.A. GROUPS

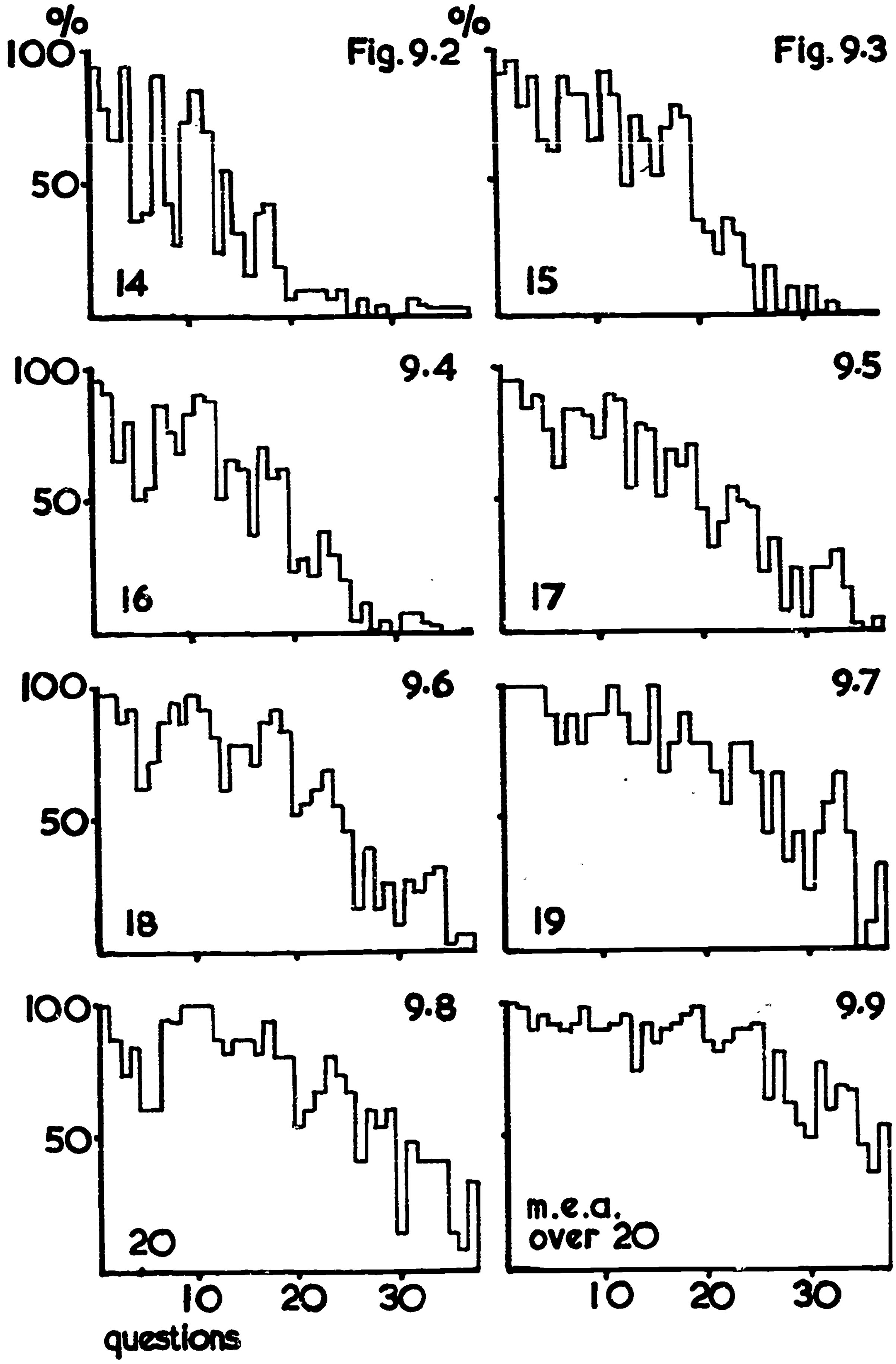
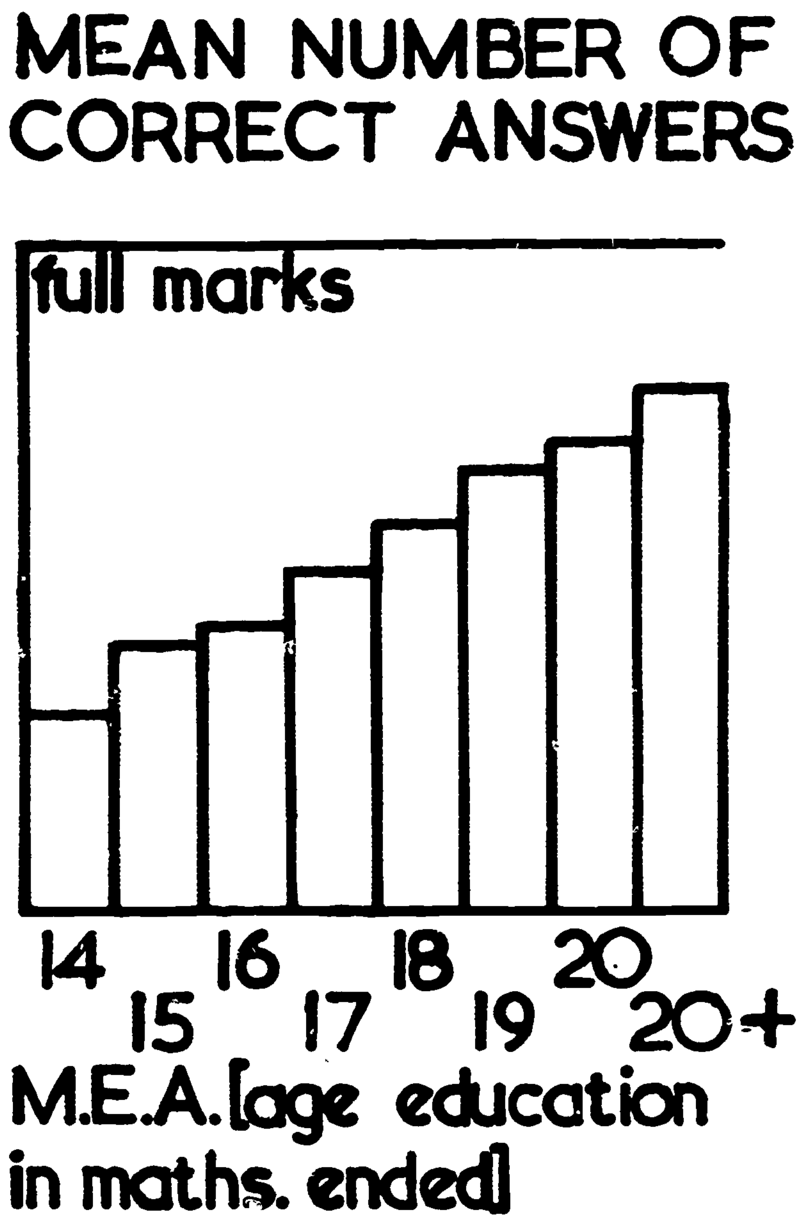


Fig. 9·10


MEAN NUMBER OF CORRECT ANSWERS
full marks
14
15
16
17
18
19
20
20+
M.E.A. [age education in maths. ended]

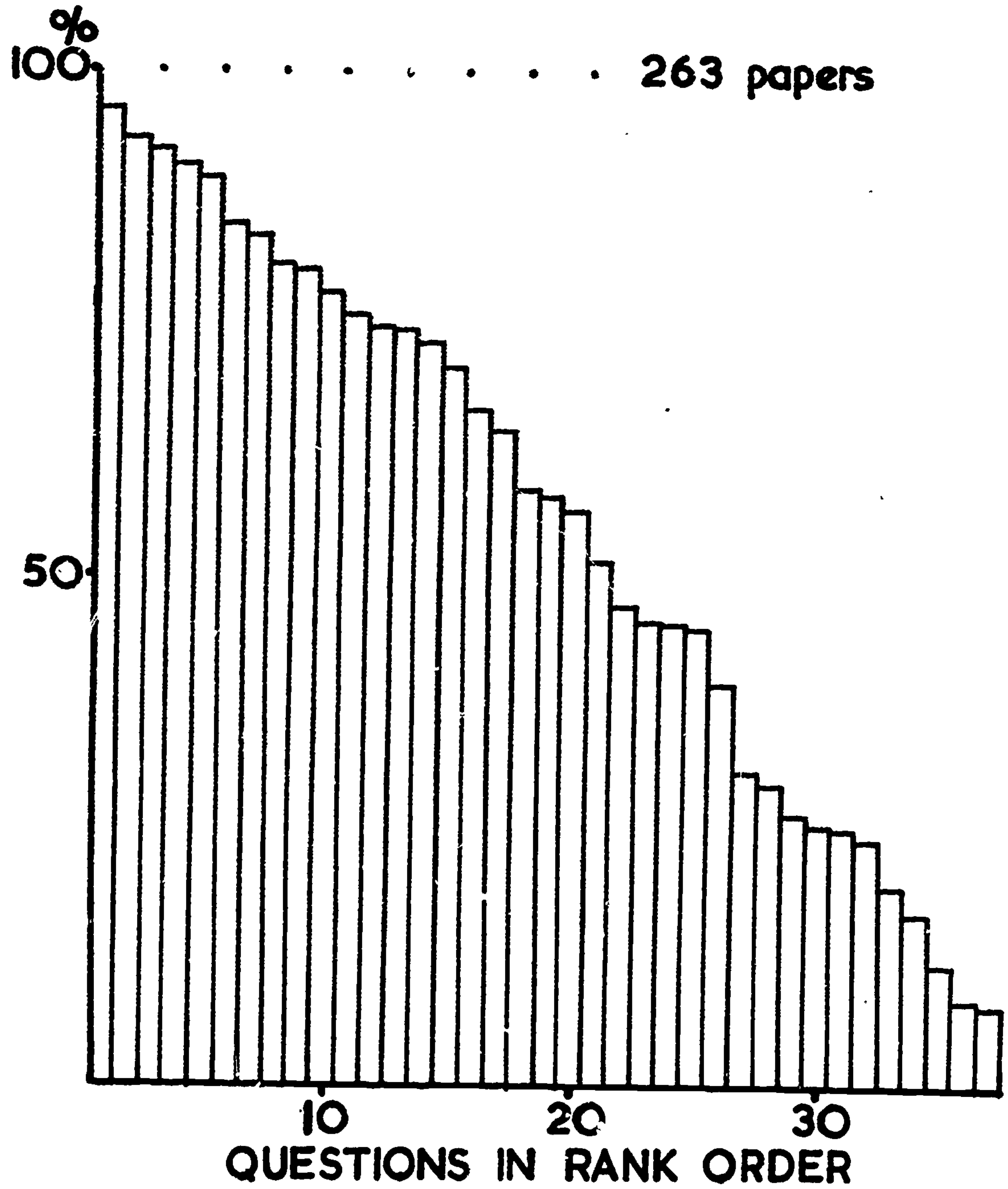
# Table 9.11

## CORRECT ANSWERS TO QUESTIONS - AS EXPECTED AND AS FOUND

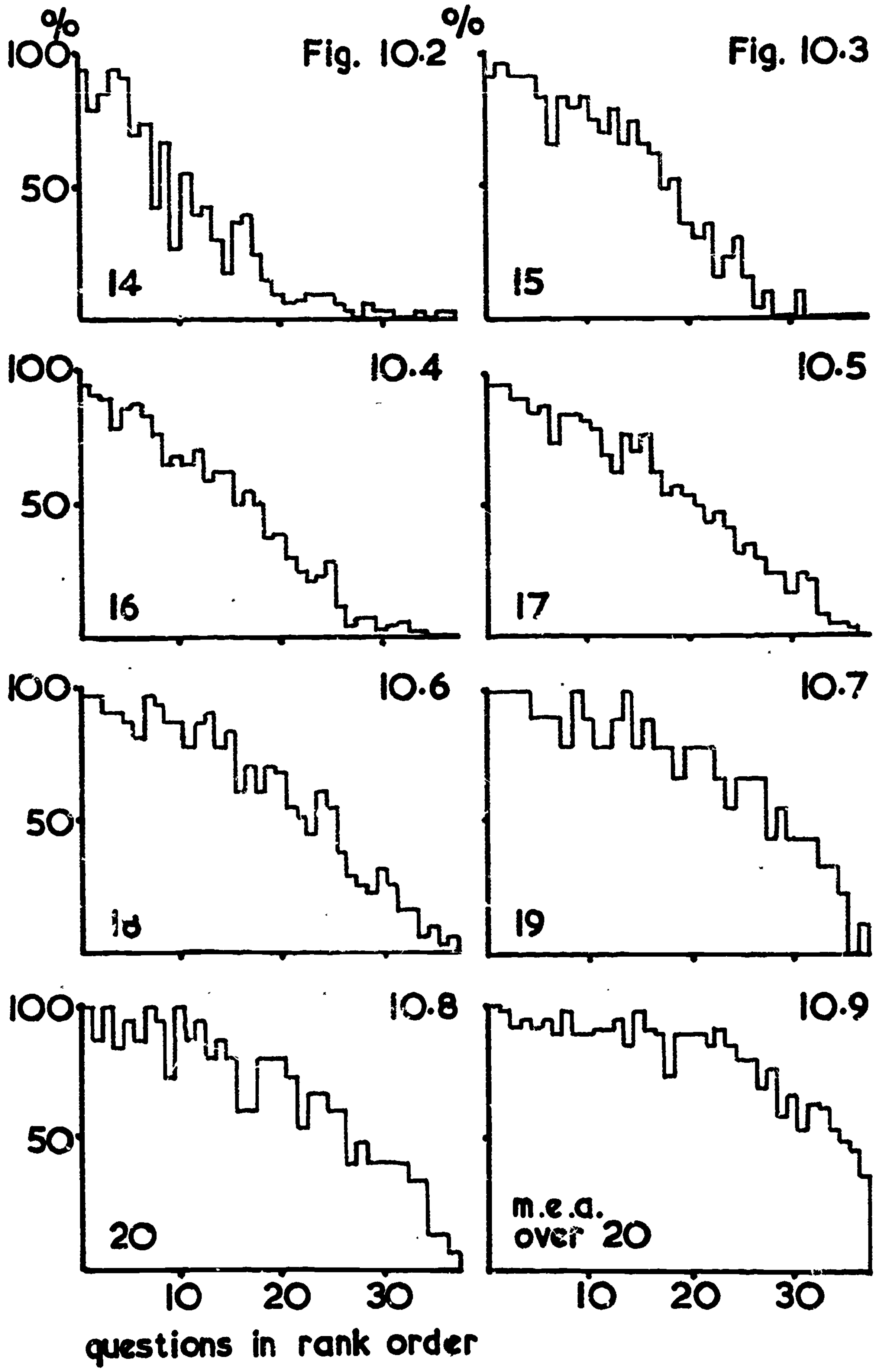
| Number of Question | Type of Question | Position in Rank Order |
|---|---|---|
| | **CORRECTLY ANSWERED** | |
| (a) MUCH MORE THAN EXPECTED | | |
| 11 | Problem in vulgar fractions | 3 |
| 12 | Problem in division of decimals | 6 |
| (b) MORE THAN EXPECTED | | |
| 17 | First degree equation problem | 12 |
| 18 | Decimal and percentage problem | 13 |
| 19 | Simultaneous equation problem | 14 |
| 31 | Graph of first degree equation | 27 |
| 33 | Graph of ellipse | 28 |
| 34 | Terminology andgeometry of ellipse | 30 |
| (c) SLIGHTLY MORE THAN EXPECTED | | |
| 7 | Mechanical subtraction of decimals | 5 |
| 10 | Unitary method | 7 |
| 14 | Square root (possible by factors) | 11 |
| 23 | Mechanical use of fractional indices | 20 |
| 24 | Use and meaning of fractional indices | 21 |
| 32 | Vector addition | 29 |
| 37 | Complex numbers | 34 |
| (d) AS EXPECTED | | |
| 1 | Long division | 1 |
| 2 | Mechanical mensuration (volume) | 2 |
| 4 | Mechanical addition of fractions | 4 |
| 8 | Mechanical first degree equation | 8 |
| 9 | Euclid (viz: state Pythagoras) | 10 |
| 15 | Linear graph from tabulated values | 14 |
| 22 | Simple problem in triginometry | 23 |
| 25 | Meaning of logarithms | 25 |
| 27 | Mechanical differentiation | 26 |
| 35 | Exponential decrease | 36 |
| 36 | Mechanical differential equation | 37 |
| (e) SLIGHTLY LESS THAN EXPECTED | | |
| 16 | Negative numbers | 19 |
| 20 | Quadratic equation problem | 22 |
| 21 | Geometry problem | 24 |
| 29 | Mechanical integration | 31 |
| (f) LESS THAN EXPECTED | | |
| 3 | Mensuration (surface area) | 9 |
| 13 | Mensuration (circumference and area of circle) | 18 |
| 26 | Circular measure (arc and radius) | 32 |
| 28 | Problem in differential calculus | 33 |
| 30 | Problem in definite integration | 35 |
| (g) MUCH LESS THAN EXPECTED | | |
| 5 | Prime factors | 16 |
| 6 | Highest common factor | 17 |

Fig. 10.1

CORRECT ANSWERS TO QUESTIONS



# CORRECT ANSWERS BY M.E.A. GROUPS



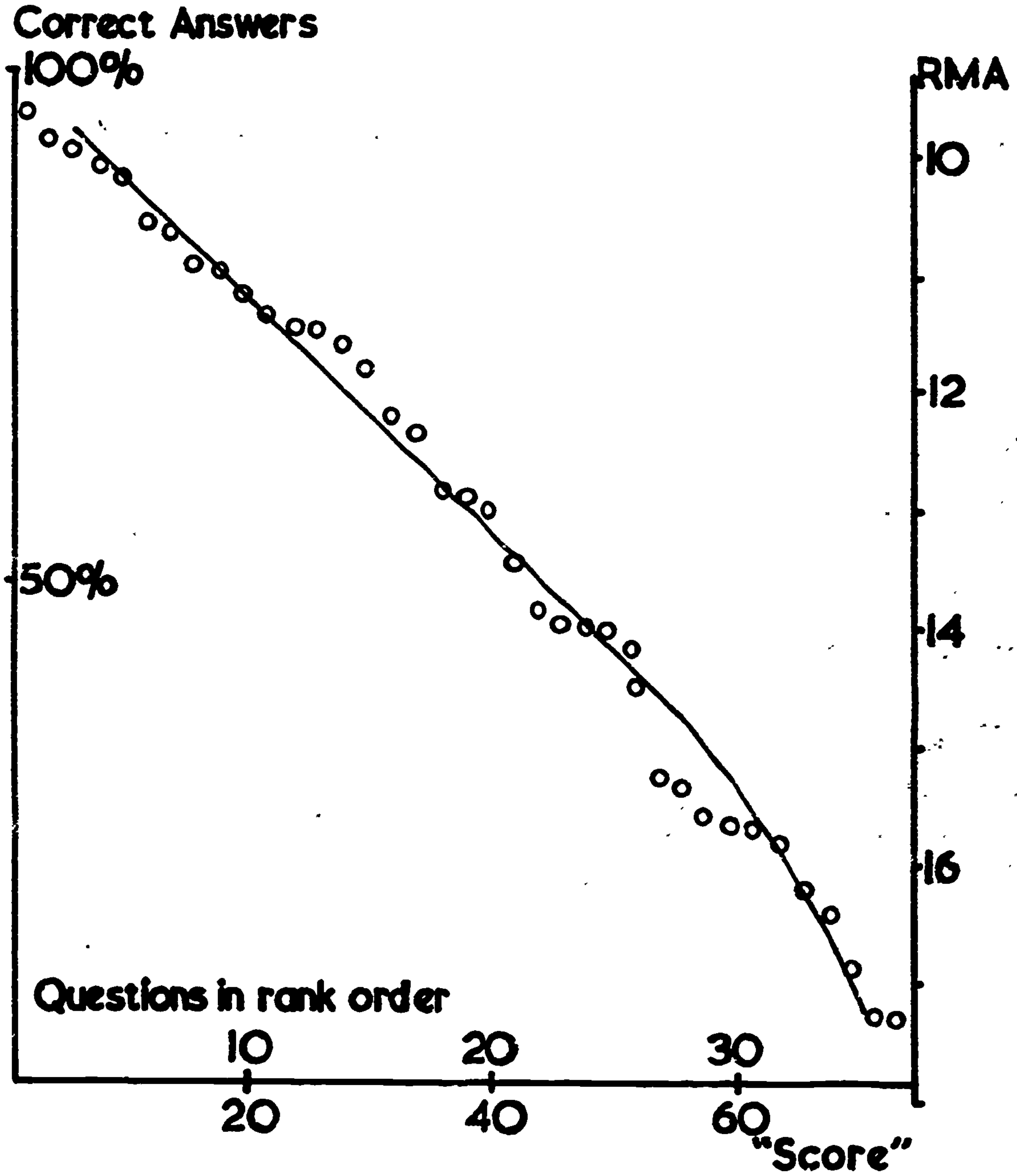
# Table 10.10

## SCORE AND RMA SCALE

| (1) SCORE | (2) RMA | (1) SCORE | (2) RMA | (1) SCORE | (2) RMA |
|---|---|---|---|---|---|
| 6 | 9.8 | 28 | 12.0 | 50 | 14.2 |
| 8 | 10.0 | 30 | 12.2 | 52 | 14.4 |
| 10 | 10.2 | 32 | 12.4 | 54 | 14.6 |
| 12 | 10.4 | 34 | 12.6 | 56 | 14.8 |
| 14 | 10.6 | 36 | 12.8 | 53 | 15.1 |
| 16 | 10.8 | 38 | 13.0 | 60 | 15.4 |
| 18 | 11.0 | 40 | 13.2 | 62 | 15.7 |
| 20 | 11.2 | 42 | 13.4 | 64 | 16.0 |
| 22 | 11.4 | 44 | 13.6 | 66 | 16.3 |
| 24 | 11.6 | 46 | 13.8 | 68 | 16.7 |
| 26 | 11.8 | 48 | 14.0 | 70 | 17.2 |

Fig. 10·11

Comparison: RMA Scale (line) and total Correct Answers to Questions (points as in Fig. 10·1)



## Table II.1

RESIDUAL MATHEMATICAL AGE & RETENTION vs. SEX
Sample & Control.

(Sample and control groups shown as 100%. Actual numbers in row f)

| | | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|---|
| | | SAMPLE | | | "NORMALISED" SAMPLE | | | CONTROL | | |
| | | MALES | FEMALES | ALL | MALES | FEMALES | ALL | MALES | FEMALES | ALL |
| RMA | | | | | | | | | | |
| (a) | UNDER 11 | 3 | 24 | 11 | 5 | 45 | 28 | 25 | 32 | 30 |
| (b) | 11 - 12 | 31 | 42 | 35 | 52 | 43 | 47 | 37 | 53 | 48 |
| (c) | 13 - 14 | 41 | 18 | 32 | 36 | 10 | 21 | 38 | 16 | 22 |
| (d) | 15 - 16 | 16 | 10 | 14 | 2 | 2 | 2 | 0 | 0 | 0 |
| (e) | OVER 16 | 9 | 7 | 8 | 5 | 0 | 2 | 0 | 0 | 0 |
| (f) | 100% | 159 | 104 | 263 | | | | 8 | 19 | 27 |
| RETENTION | | | | | | | | | | |
| (g) | UNDER 60 | 1 | 4 | 2 | 2 | 7 | 5 | 0 | 11 | 7 |
| (h) | 60 - 69 | 9 | 22 | 14 | 21 | 32 | 27 | 38 | 42 | 41 |
| (i) | 70 - 79 | 30 | 44 | 36 | 33 | 47 | 41 | 25 | 26 | 26 |
| (j) | 80 - 89 | 47 | 23 | 37 | 35 | 14 | 23 | 25 | 21 | 22 |
| (k) | OVER 89 | 13 | 7 | 11 | 9 | 0 | 4 | 12 | 0 | 4 |

# Table II.2

## MEDIAN RMA RETENTION vs. SEX.

| | | (1) | (2) | (3) | (4) |
|---|---|---|---|---|---|
| | | MEDIAN RMA | | MEDIAN RETENTION | |
| | | RAW SAMPLE | NORMALISED SAMPLE | RAW SAMPLE | NORMALISED SAMPLE |
| (a) | MALES | 13.8 | 12.7 | 81 | 77 |
| (b) | FEMALES | 12.2 | 11.2 | 74 | 71 |
| (c) | ALL | 13.3 | 11.9 | 79 | 73 |

# Table II.3

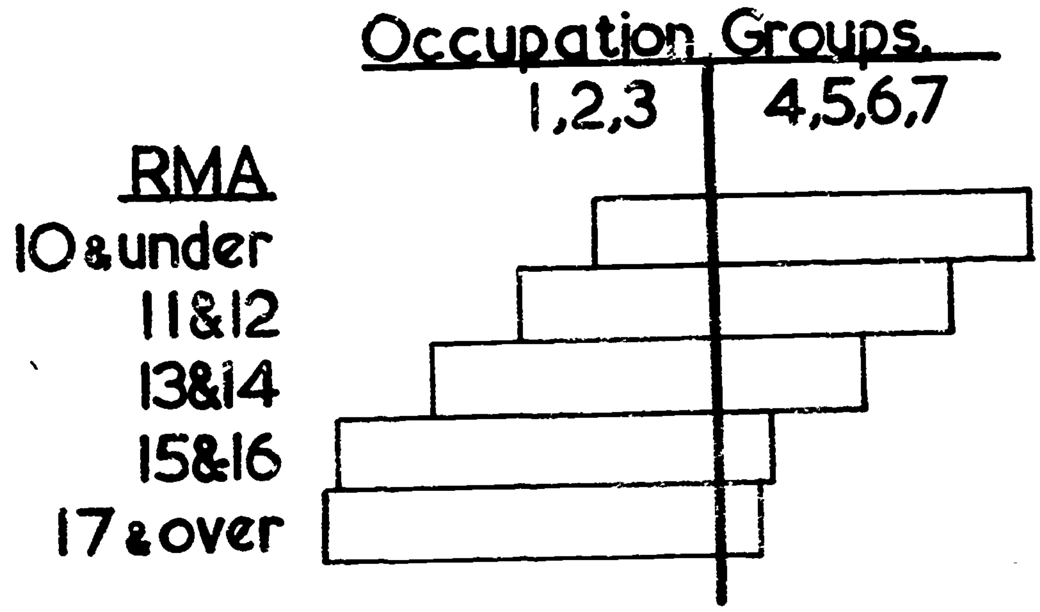
## RESIDUAL MATHEMATICAL AGE & RETENTION vs. OCCUPATION GROUP: Sample & Control.

(Percentages except columns 1 and 6 and row f)

| | | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|---|
| | | SAMPLE | | | CONTROL | | |
| | | NUMBERS | OCCUPATION GROUPS | | OCCUPATION GROUPS | | NUMBERS |
| | | | 1,2,3 | 4,5,6,7 | 1,2,3 | 4,5,6,7 | |
| | **RMA** | | | | | | |
| (a) | 10 & UNDER | 30 | 5 | 21 | 30 | 29 | 8 |
| (b) | 11 & 12 | 93 | 27 | 47 | 30 | 59 | 13 |
| (c) | 13 & 14 | 83 | 34 | 26 | 40 | 12 | 6 |
| (d) | 15 & 16 | 36 | 21 | 4 | 0 | 0 | 0 |
| (e) | 17 & OVER | 21 | 13 | 2 | 0 | 0 | 0 |
| (f) | 100% | 263 | 155 | 105 | 10 | 17 | 27 |
| | **RETENTION** | | | | | | |
| (g) | 59 & UNDER | 7 | 3 | 2 | 20 | 0 | 2 |
| (h) | 60 - 69 | 37 | 13 | 17 | 10 | 59 | 11 |
| (i) | 70 - 79 | 94 | 35 | 36 | 30 | 23 | 7 |
| (j) | 80 - 89 | 96 | 33 | 41 | 30 | 18 | 6 |
| (k) | 90 & OVER | 29 | 16 | 4 | 10 | 0 | 1 |

Fig. II-4

# RMA & OCCUPATION
(RMA GROUPS OF EQUAL SIZE)



## Table 11.5

RESIDUAL MATHEMATICAL AGE & RETENTION vs. AGE : Samplë & 'Normalised' Sample.

(Age groups shown as 100%. Actual numbers in column 1 and row f)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| | NUMBERS | SAMPLE AGE | | | | "NORMALISED" SAMPLE AGE | | | |
| | | UNDER 35 | 35-54 | OVER 54 | ALL | UNDER 35 | 35-54 | OVER 54 | ALL |
| RMA | | | | | | | | | |
| (a) UNDER 11 | 30 | 12 | 8 | 18 | 11 | 34 | 17 | 35 | 28 |
| (b) 11 - 12 | 93 | 26 | 42 | 41 | 35 | 37 | 50 | 55 | 47 |
| (c) 13 - 14 | 83 | 33 | 29 | 33 | 32 | 23 | 28 | 10 | 21 |
| (d) 15 - 16 | 36 | 19 | 12 | 6 | 14 | 3 | 3 | 0 | 2 |
| (e) OVER 16 | 21 | 11 | 8 | 2 | 8 | 3 | 3 | 0 | 2 |
| (f) 100% | 263 | 101 | 113 | 49 | 263 | (35) | (36) | (29) | (100) |
| RETENTION | | | | | | | | | |
| (g) UNDER 60 | 7 | 1 | 3 | 4 | 3 | 3 | 5 | 3 | 5 |
| (h) 60 - 69 | 37 | 17 | 11 | 16 | 14 | 37 | 17 | 28 | 27 |
| (i) 70 - 79 | 94 | 31 | 35 | 47 | 35 | 37 | 31 | 59 | 41 |
| (j) 80 - 89 | 96 | 39 | 40 | 23 | 37 | 17 | 42 | 10 | 23 |
| (k) OVER 89 | 29 | 12 | 11 | 10 | 11 | 6 | 5 | 0 | 4 |

# Table II.6

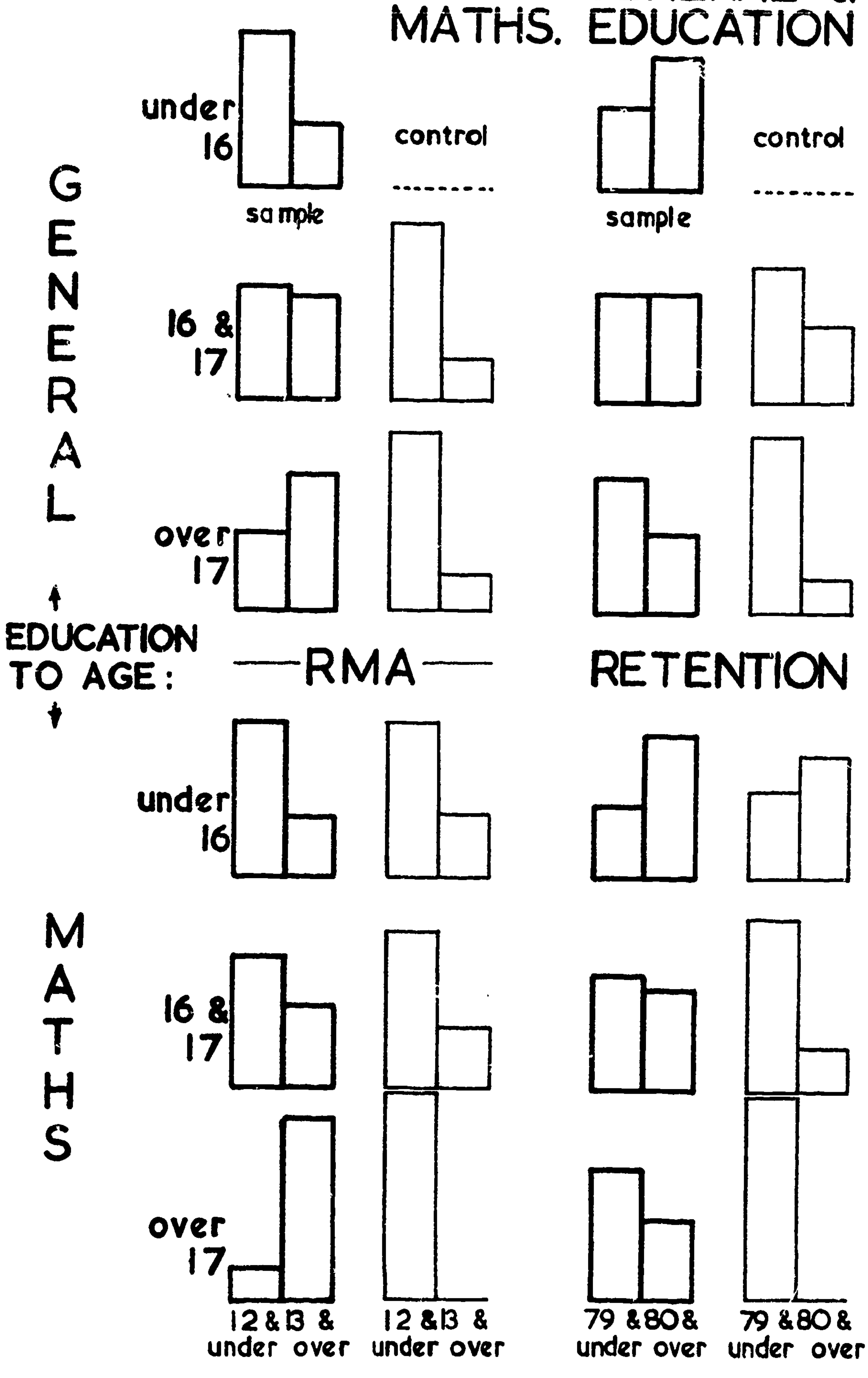
RESIDUAL MATHEMATICAL AGE & RETENTION vs. GENERAL EDUCATION & MATHEMATICS EDUCATION: Sample and Control Groups.

(Education groups shown as 100% with sizes of groups in rows i and r)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| | AGE GENERAL EDUCATION ENDED | | | | AGE MATHS. EDUCATION ENDED | | | |
| | UNDER 16 | 16 & 17 | 18 & 19 | 20 & OVER | UNDER 16 | 16 & 17 | 18 & 19 | 20 & OVER |
| **SAMPLE** | | | | | | | | |
| **RMA** | | | | | | | | |
| (a) 10 & UNDER | 21 | 13 | 9 | 6 | 23 | 13 | 5 | 0 |
| (b) 11 & 12 | 50 | 39 | 32 | 26 | 50 | 48 | 15 | 11 |
| (c) 13 & 14 | 23 | 37 | 38 | 29 | 25 | 31 | 50 | 24 |
| (d) 15 & OVER | 6 | 11 | 21 | 39 | 2 | 8 | 30 | 65 |
| **RETENTION** | | | | | | | | |
| (e) 69 & UNDER | 8 | 17 | 18 | 21 | 4 | 18 | 18 | 28 |
| (f) 70 - 79 | 31 | 33 | 50 | 39 | 30 | 36 | 45 | 33 |
| (g) 80 - 89 | 46 | 44 | 26 | 26 | 46 | 37 | 27 | 32 |
| (h) 90 & OVER | 15 | 6 | 6 | 14 | 20 | 9 | 10 | 7 |
| (i) 263 = 100% | 48 | 84 | 34 | 97 | 56 | 113 | 40 | 54 |
| **CONTROL** | | | | | | | | |
| **RMA** | | | | | | | | |
| (j) 10 & UNDER | 33 | 36 | 38 | 0 | 43 | 20 | 40 | 0 |
| (k) 11 & 12 | 0 | 46 | 50 | 80 | 29 | 53 | 60 | 0 |
| (l) 13 & 14 | 67 | 18 | 12 | 20 | 29 | 27 | 0 | 0 |
| (m) 15 & OVER | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **RETENTION** | | | | | | | | |
| (n) 69 & UNDER | 33 | 36 | 75 | 40 | 43 | 33 | 100 | 0 |
| (o) 70 - 79 | 33 | 27 | 12 | 40 | 0 | 47 | 0 | 0 |
| (p) 80 - 89 | 33 | 36 | 12 | 20 | 29 | 20 | 0 | 0 |
| (q) 90 & OVER | 0 | 0 | 0 | 0 | 29 | 0 | 0 | 0 |
| (r) 27 = 100% | 3 | 11 | 8 | 5 | 7 | 15 | 5 | 0 |

Fig. II·7

# RMA. & RETENTION vs. GENERAL & MATHS. EDUCATION



# Table II.8

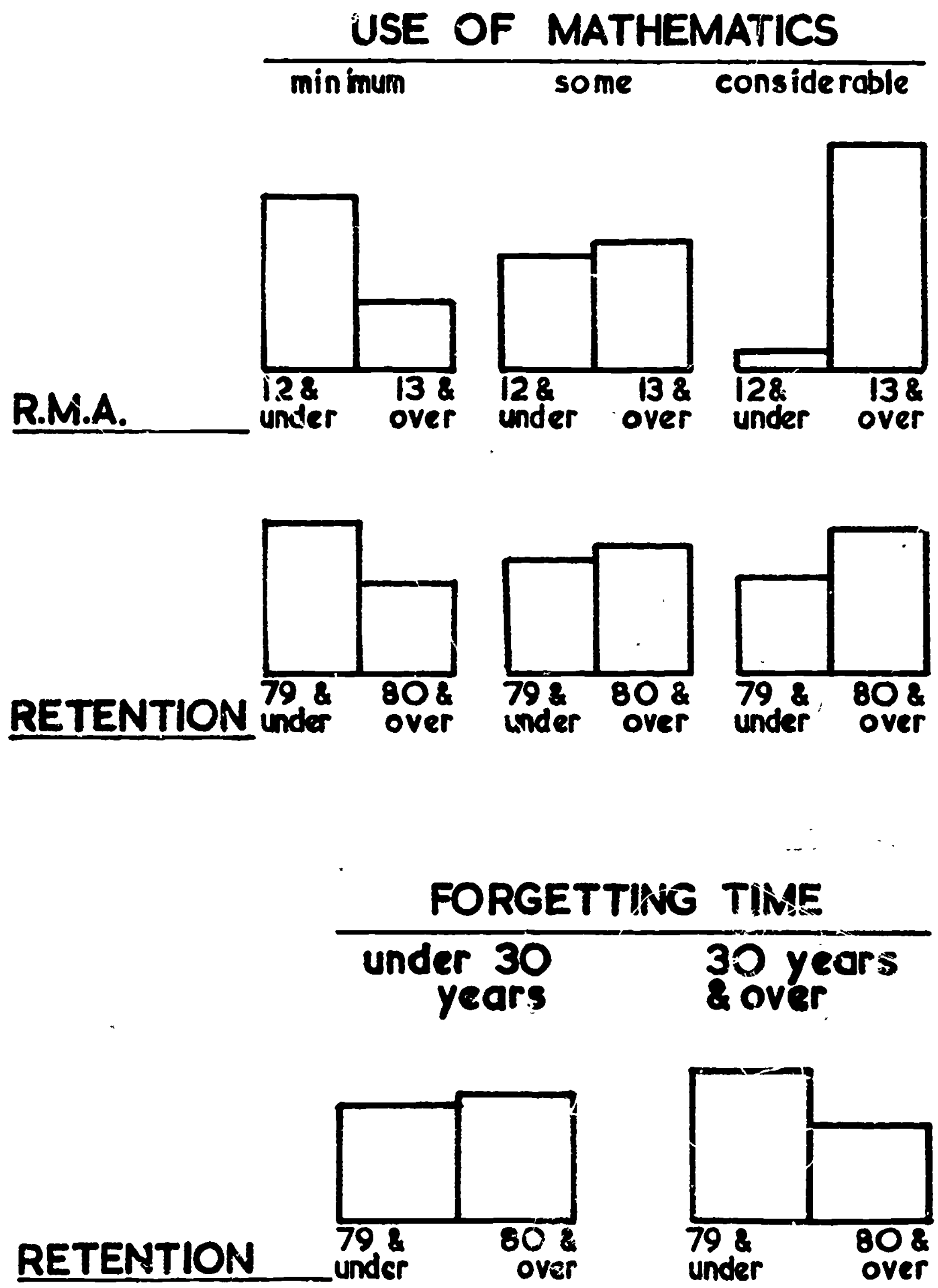
## RESIDUAL MATHEMATICAL AGE & RETENTION vs. USE OF MATHEMATICS and 'FORGETTING TIME'

(Use and forgetting time groups shown as 100%; actual numbers in column 4 and row j)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| | USE OF MATHS. | | | NUMBERS | "FORGETTING TIME" IN YEARS | | | | |
| | Minimum | Some | Considerable | | UNDER 10 | 10-19 | 20-29 | 30-39 | OVER 39 |
| RMA | | | | | | | | | |
| (a) UNDER 11 | 21 | 7 | 0 | 30 | | | | | |
| (b) 11 - 12 | 50 | 40 | 8 | 92 | | | | | |
| (c) 13 - 14 | 24 | 43 | 32 | 83 | | | | | |
| (d) 15 - 16 | 4 | 9 | 32 | 35 | | | | | |
| (e) OVER 16 | 1 | 1 | 28 | 21 | | | | | |
| RETENTION | | | | | | | | | |
| (f) UNDER 70 | 26 | 10 | 9 | 44 | 19 | 21 | 11 | 14 | 22 |
| (g) 70 - 79 | 37 | 37 | 31 | 92 | 29 | 30 | 35 | 45 | 41 |
| (h) 80 - 89 | 33 | 43 | 37 | 96 | 44 | 34 | 44 | 31 | 24 |
| (i) OVER 89 | 4 | 10 | 23 | 29 | 8 | 15 | 10 | 10 | 13 |
| (j) 100% | 117 | 70 | 74 | 261 | 59 | 53 | 63 | 49 | 37 |

Fig. 11·9

RMA & RETENTION vs. 'USE OF MATHS' & 'FORGETTING TIME'



# Table 12.1

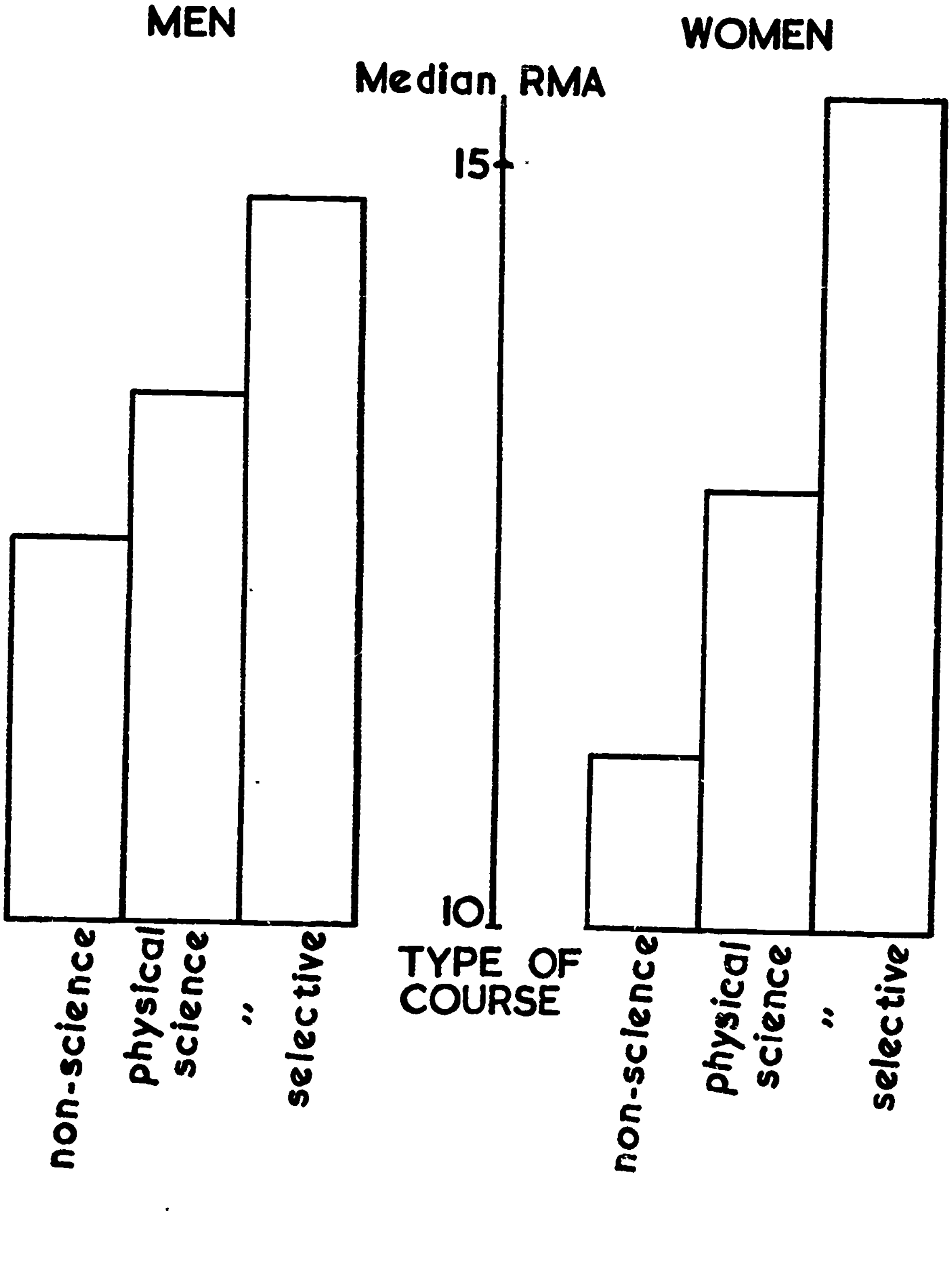
## RMA & RETENTION vs. SUBJECT (with subdivision by Age & Sex) & LOCATION OF COURSE.

(Subject, sex, age and location groups (rows) shown as 100% with actual numbers in column 6)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | RMA | | | | | 100% | RETENTION | | | | |
| | UNDER 11 | 11 & 12 | 13 & 14 | 15 & 16 | OVER 16 | | UNDER 60 | 60/69 | 70/79 | 80/89 | OVER 89 |
| **SUBJECT** | | | | | | | | | | | |
| **PHYSICAL SCIENCE (SELECTIVE)** | | | | | | | | | | | |
| SEX | | | | | | | | | | | |
| (a) M | 0 | 2 | 53 | 27 | 18 | 51 | 0 | 6 | 20 | 47 | 27 |
| (b) F | 0 | 7 | 36 | 29 | 29 | 14 | 0 | 0 | 29 | 36 | 36 |
| AGE | | | | | | | | | | | |
| (c) UNDER 35 | 0 | 0 | 48 | 32 | 21 | 34 | 0 | 9 | 24 | 50 | 18 |
| (d) 35 - 54 | 0 | 9 | 39 | 26 | 26 | 23 | 0 | 0 | 13 | 48 | 39 |
| (e) OVER 54 | 0 | 0 | 87 | 13 | 0 | 8 | 0 | 0 | 37 | 13 | 50 |
| (f) ALL | 0 | 3 | 49 | 28 | 20 | 65 | 0 | 5 | 22 | 45 | 29 |
| **PHYSICAL SCIENCE (NON-SELECTIVE)** | | | | | | | | | | | |
| SEX | | | | | | | | | | | |
| (g) M | 2 | 39 | 38 | 15 | 7 | 61 | 0 | 7 | 33 | 54 | 7 |
| (h) F | 6 | 45 | 23 | 19 | 6 | 31 | 3 | 16 | 39 | 36 | 6 |
| AGE | | | | | | | | | | | |
| (i) UNDER 35 | 3 | 39 | 27 | 21 | 9 | 33 | 0 | 9 | 30 | 51 | 9 |
| (j) 35 - 54 | 4 | 47 | 31 | 13 | 4 | 45 | 2 | 11 | 38 | 44 | 4 |
| (k) OVER 54 | 0 | 29 | 50 | 14 | 7 | 14 | 0 | 7 | 36 | 50 | 7 |
| (l) ALL | 3 | 41 | 33 | 16 | 7 | 92 | 1 | 10 | 35 | 48 | 7 |
| **BIOLOGICAL SCIENCE** | | | | | | | | | | | |
| SEX | | | | | | | | | | | |
| (m) M | 17 | 58 | 17 | 8 | 0 | 12 | 8 | 8 | 58 | 25 | 0 |
| (n) F | 17 | 75 | 8 | 0 | 0 | 12 | 0 | 33 | 58 | 0 | 8 |
| AGE | | | | | | | | | | | |
| (o) UNDER 35 | 33 | 33 | 33 | 0 | 0 | 3 | 0 | 33 | 33 | 0 | 33 |
| (p) 35 - 54 | 12 | 69 | 12 | 6 | 0 | 16 | 6 | 12 | 69 | 12 | 0 |
| (q) OVER 54 | 20 | 80 | 0 | 0 | 0 | 5 | 0 | 40 | 40 | 20 | 0 |
| (r) ALL | 17 | 67 | 12 | 4 | 0 | 24 | 4 | 21 | 58 | 12 | 4 |
| **NON-SCIENCE** | | | | | | | | | | | |
| SEX | | | | | | | | | | | |
| (s) M | 7 | 57 | 32 | 0 | 4 | 28 | 4 | 25 | 32 | 32 | 7 |
| (t) F | 48 | 43 | 9 | 0 | 0 | 44 | 7 | 32 | 48 | 14 | 0 |
| AGE | | | | | | | | | | | |
| (u) UNDER 35 | 40 | 40 | 20 | 0 | 0 | 25 | 4 | 40 | 40 | 12 | 4 |
| (v) 35 - 54 | 20 | 52 | 24 | 0 | 4 | 25 | 8 | 20 | 28 | 40 | 4 |
| (w) OVER 54 | 36 | 55 | 9 | 0 | 0 | 22 | 5 | 27 | 59 | 9 | 0 |
| (x) ALL | 32 | 49 | 18 | 0 | 1 | 72 | 6 | 29 | 42 | 21 | 3 |
| **ALL SUBJECTS** | | | | | | | | | | | |
| (y) ALL GROUPS | 11 | 35 | 32 | 14 | 8 | 253 | 2 | 14 | 36 | 37 | 11 |
| **LOCATIONS OF COURSES** | | | | | | | | | | | |
| (z) CITY | 6 | 28 | 36 | 19 | 11 | 152 | 1 | 9 | 31 | 42 | 17 |
| (A) SUBURB | 17 | 46 | 26 | 9 | 2 | 47 | 2 | 21 | 40 | 31 | 6 |
| (B) COUNTY | 24 | 51 | 18 | 2 | 5 | 54 | 6 | 27 | 46 | 21 | 0 |

Fig. 12.2

MEDIAN RMA FOR MEN & WOMEN IN VARIOUS COURSES



## GLOSSARY

Short explanations of terms used frequently in the text are given here for convenient reference but more complete definitions and discussions of their validity are given in the chapters indicated.

| | |
|---|---|
| NORMALISING. | Data relating to groups of adults in the sample drawn from different types of extra-mural course has been NORMALISED by "weighting" it to reflect the proportions in which these types of course occur in Extra-Mural studies as a whole (i.e. by multiplying by numerical factors derived from Adult Education statistics). (Chapter 6). |
| MATHEMATICS EDUCATION AGE | (MEA): is the age to which the adult received formal juvenile instruction in mathematics at school or college. (Chapter 10). |
| RESIDUAL MATHEMATICAL AGE | (RMA): If an adult retains mathemtical knowledge (as indicated by the Test) equivalent to that which he was taught at (for example) age 15, he is said to have RMA 15. (Chapter 10). |
| RETENTION: | is the fraction (expressed as a percentage) which an adult retains of the mathematics he was taught when young. (Chapter 10). |

This investigation, originally planned as a single item in Harry Frost's responsibility for research into ways of conveying an appreciation of scientific ideas to non-scientists, produced additional information and ideas with implications for the teaching and examining of mathematics and for adult education which are presented and discussed in this Paper.

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